24-650 Applied Finite Element Analysis

Homework No 1

Steady state thermal analysis of a coffee cup

Ignacio Cordova

The objective of this assignment was to do a steady state thermal analysis of a coffee cup filled with boiling water in Ansys. The coffee cup was created using SpaceClaim (Figure 1) and then imported to Ansys Workbench. The **wall thickness** of the cup is **3 mm**.

1. Setup

The first step was to create a new project in Ansys Workbench and set a Steady-State Thermal module. Using the Engineering Data option, I created three different materials with three different **Isotropic Thermal Conductivity** values as shown in the Table 1.

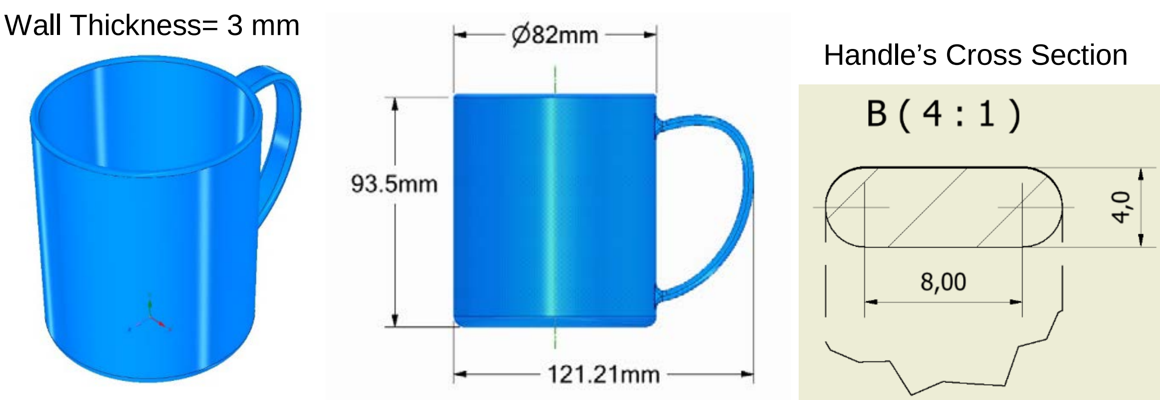

Figure 1: Coffee Cup Design in SpaceClaim

Material	*k* [W/mºC]
Cast Iron	60
Stainless Steel	17
Glass	1

Table 1: Materials created for the analysis

Then, I imported the geometry to Ansys Mechanical and I created the **default mesh** which consisted in **7,745 nodes** and **3,891 elements**. This is shown in the Figure 2.

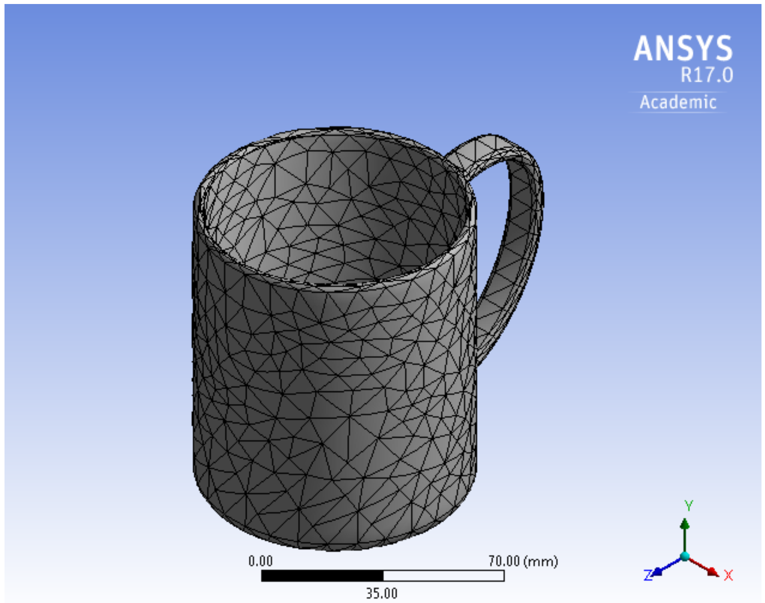

Figure 2: Default Mesh

With the mesh ready, I started creating the boundary conditions for the problem. These are:

- Simplified convection for stagnant air, $T_f = 22\ ℃$ for the outer surface (Figure A.1)
- Simplified convection for stagnant water, $T_f = 100\ ℃$ for the inner surface (Figure A.2)
- Temperature $T^* = 22\ ℃$ for the bottom surface (Figure A.3)

With those conditions, the simulation was done for Cast Iron, Stainless Steel and Glass. The results are shown in the next section.

2. Results and Analysis

The results for temperature and total heat flux are shown for the three materials in the Appendix (Figure A.4 to Figure A.8). For this study, it is only important to know the average temperature on the handle, because that is the part that is going to be in contact with the skin. Taking four similar points on the handle for each material and calculating the average, we can get a better understanding of the temperature that a person would feel by grabbing the handle. These results are shown in the Table 2.

Material	Handle Average Temperature [ºC]
Cast Iron	82.99
Stainless Steel	79.00
Glass	37.84

Table 2: Average Temperature at the Handle

According to the document *A New Approach to Defining Human Touch Temperature Standards*[1] , the upper limit temperature for contact with hot objects should be around $\mathbf{44}$℃. Considering that value, my cup design doesn't work very well for Cast Iron and Stainless Steel. For both materials, the average temperature at the handle is at least **80% hotter** than the recommended temperature. Only for the Glass does my design work well, with an average temperature of **13% less** than the recommended one. This makes sense, because the isotropic thermal conductivity of the glass is very small compared to the Cast Iron or Stainless Steel. This means that the heat tends to go directly to the bottom, using the surface resting at 22 ℃ as a sink instead of going to the handle. For the cast iron and stainless steel it is different. Both materials have a high isotropic thermal conductivity, so it is easier for the heat to flow through those materials and reach the handle. It is also important to notice that I took the average of four points (Figure A.8) but the maximum temperature at the handle using Glass is $\mathbf{55,48}$ ℃**.** This means that there is a small section at the beginning of the handle that has a temperature hotter that recommended, and that section is probably touched by the index finger.

Due to those results, we can say that the material selection is extremely important in the design of a coffee cup. By using materials with a low isotropic thermal conductivity, we can have a handle that a person can actually grab. The option of using high isotropic thermal conductivity materials is only reasonable if the handle has some cover made of a low isotropic thermal conductivity material like cork, rubber, silicone, among others. It is also important to analyze if, by changing the geometry of the cup, we can have a better design with lower temperatures at the handle. For this analysis, I changed the thickness of the wall from **3 mm to 5 mm** and I updated the cross section of the handle to the one shown in Figure 3.

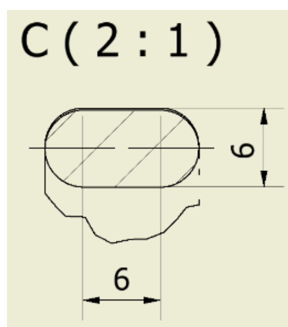

Figure 3: Updated Cross Section

The result is presented in Figure A.10. Comparing it with Figure A.4, we can see that the temperatures are pretty similar for both geometries and they only differ by less than **2%**. This means that manipulating the cup's geometry doesn't make a significant difference in the temperatures on the handle. The explanation for that is that the effect of the isotropic thermal conductivity is bigger than the effect of the geometry. Obviously, by making a huge change in the geometry the results could vary more, but the geometry of the cup probably wouldn't be realistic.

About the boundary conditions, I think they are **too conservative**. Especially because we are not considering convection between the top surface of water and the air, so for our model the heat is lost only through the cup surface and we are treating the volume of water as if it has a cover on the top,

[1] Stroud, E. U. (n.d.). *A New Approach to Defining Human Touch Temperature Standards.* NASA Johnson Space Center.

avoiding the interaction with the air. Also, because we are using a simplified convection for stagnant water at $T_f = 100\ ℃$, we are assuming that the water is always at that temperature which is not true. The correct boundary conditions would be to consider the water as a fluid that is interacting with the air and with the surface of the cup and changing its temperature. I also think that assuming the cup is completely filled with boiling water is too conservative. A normal cup is possibly filled 90%, so the inner surface of the cup should have 10% considering convection with the air and 90% with boiling water.

Another assumption that can be questioned is the Temperature $T^* = 22\ ℃$ for the bottom surface. This would it be true if the bottom surface is a really large and good heat conductor, like a big steel table, but for a normal wood table it is not completely true. Because of the low thermal conductivity of the wood (around 0.12 [W/mºC][2]), the heat transfer from the bottom of the cup to the wood is going to be slow, so the heat will get stuck in that zone heating it up a little bit and changing the conditions.

3. Conclusions

For any analysis it is really important to first understand the physics behind the problem and try to think of what it is important and what it is not. For this problem, a steady-state thermal analysis of a coffee cup was made. There is no need to do a transient analysis because of the nature of the problem. Generally, a coffee cup filled with boiling water will stay for many minutes before heating most of its body. Those many minutes are what we can use as an argument to treat this as a steady-state problem instead of a transient problem, where a short time effect is analyzed. The advantages to doing so are mainly how easy it is to set a steady-state analysis instead of a transient one. Also, the computational resources are considerably less in a steady-state analysis. Nevertheless, to obtain some reasonable results, it is important to have correct boundary conditions. For this case, the boundary conditions selected where enough to do the simulation and to get some real and reasonable results, but they were too conservative.

[2] *The Engineering Toolbox*. (n.d.). Retrieved from http://www.engineeringtoolbox.com/thermal-conductivity-d_429.html

4. References

Stroud, E. U. (n.d.). *A New Approach to Defining Human Touch Temperature Standards.* NASA Johnson Space Center.

The Engineering Toolbox. (n.d.). Retrieved from http://www.engineeringtoolbox.com/thermal-conductivity-d_429.html

5. Appendix

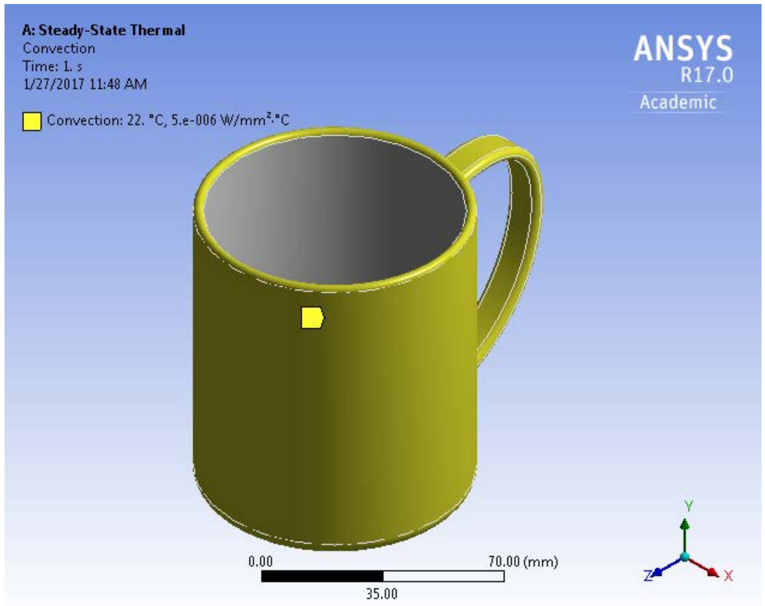

Figure A.1: BC Convection 1

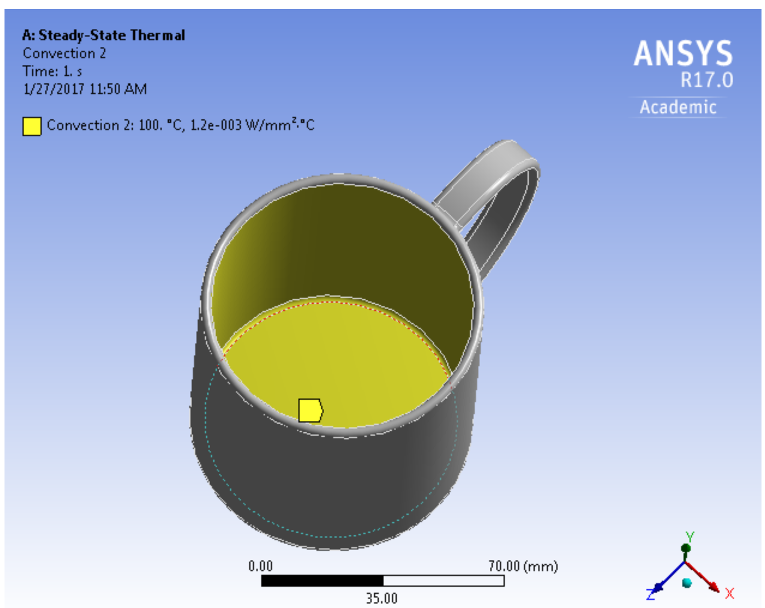

Figure A.2: BC Convection 2

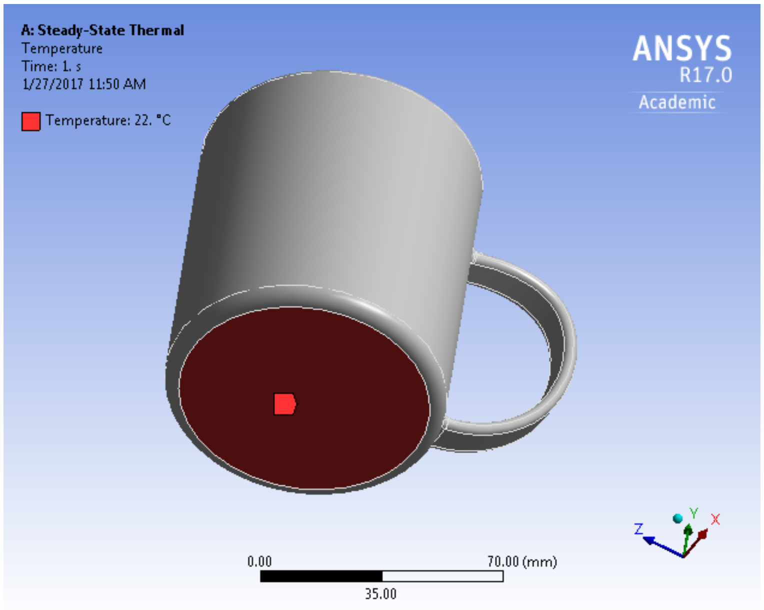

Figure A.3: BC Temperature

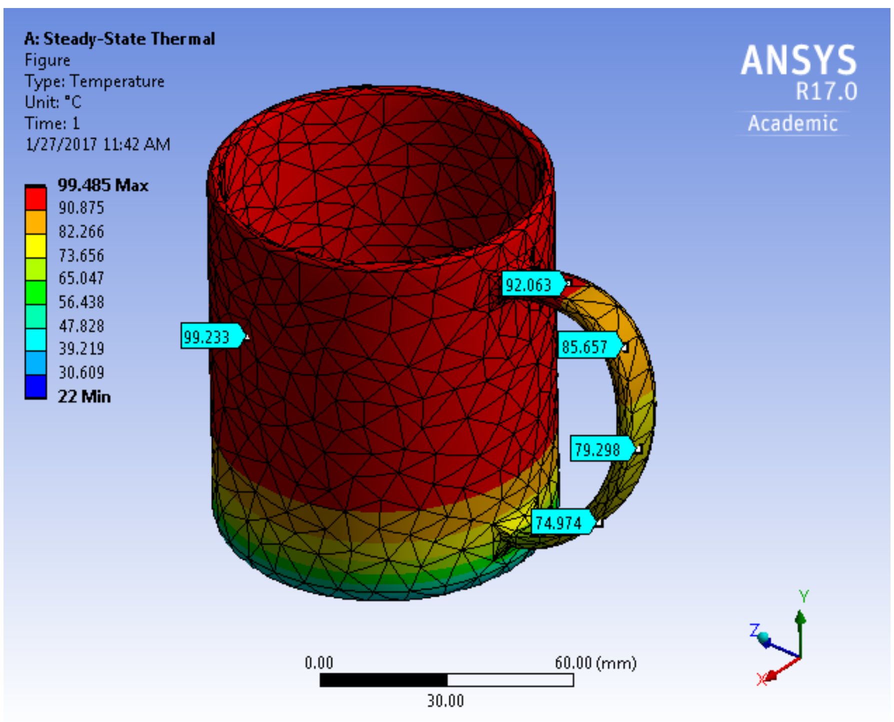

Figure A.4: Cast Iron Temperature

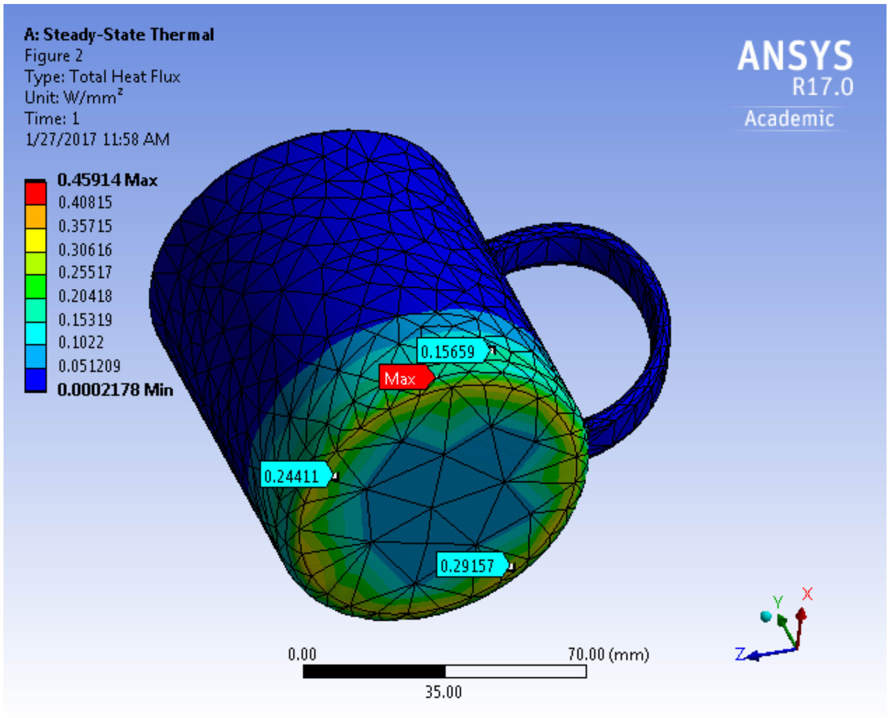

Figure A.5: Cast Iron Total Heat Flux

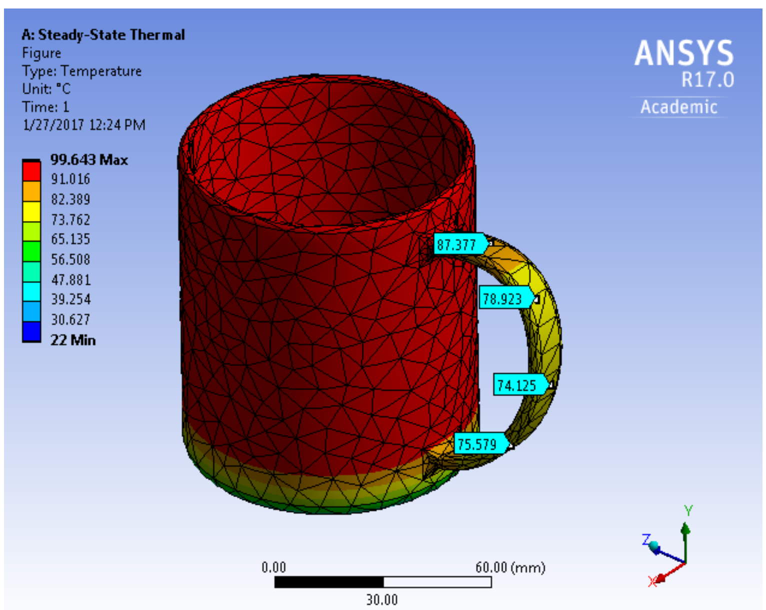

Figure A.6: Stainless Steel Temperature

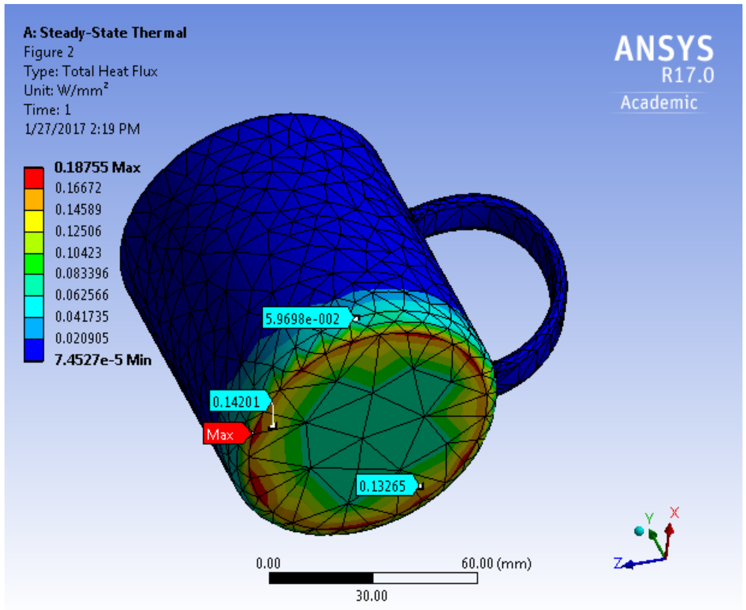

Figure A.7: Stainless Steel Total Heat Flux

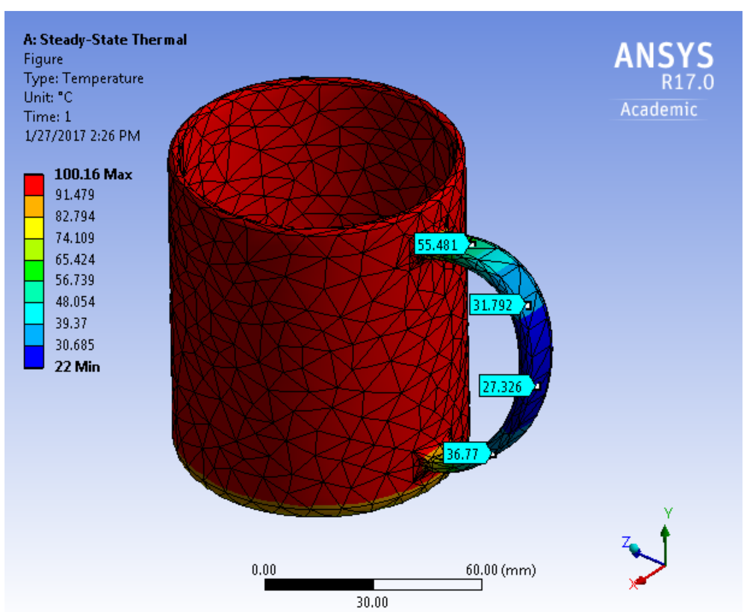

Figure A.8: Glass Temperature

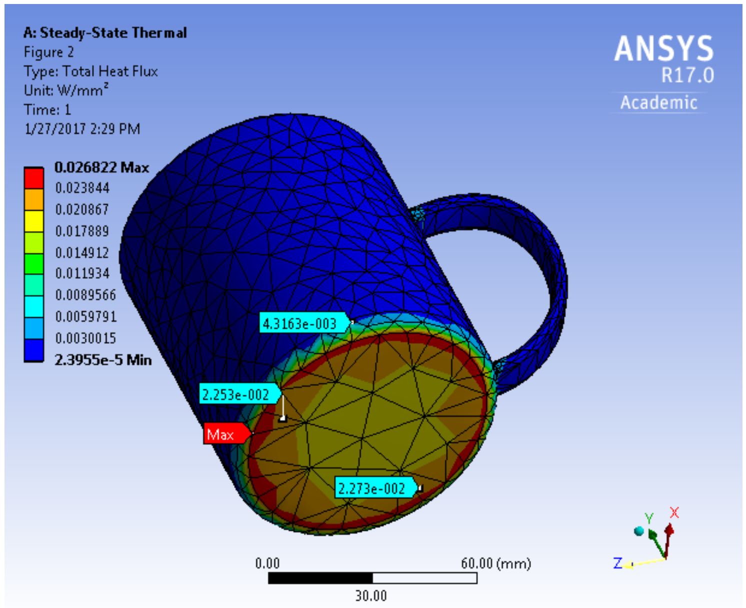

Figure A.9: Glass Total Heat Flux

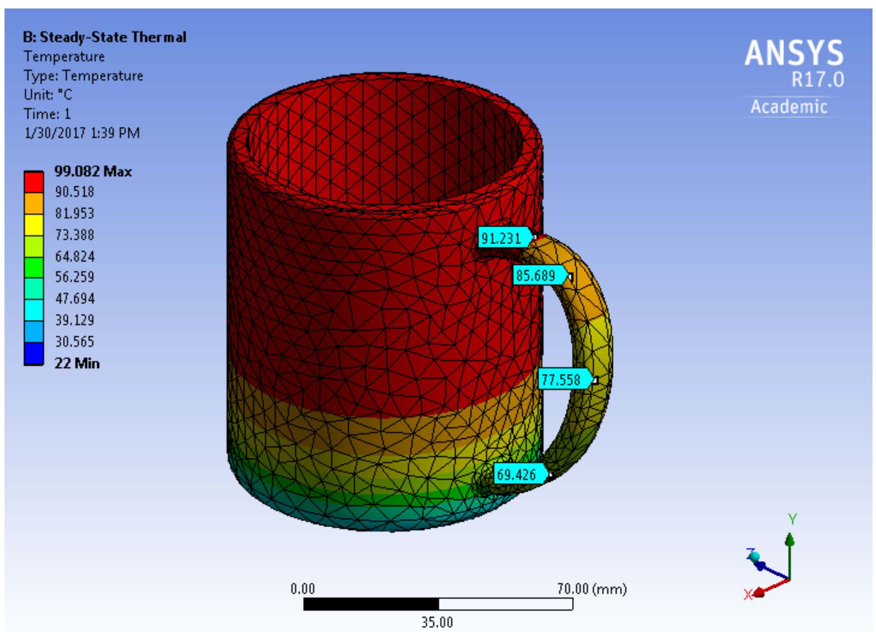

Figure A.10: Cast Iron Temperature Updated Geometry