24-650 Applied Finite Element Analysis
Homework No 12
Not-Perceptible Vibration Design of a Stairway
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The objective of this assignment was to determine if a given design of a staircase is acceptable or not in terms of perceptible vibration. The staircase is shown in Figure 1.

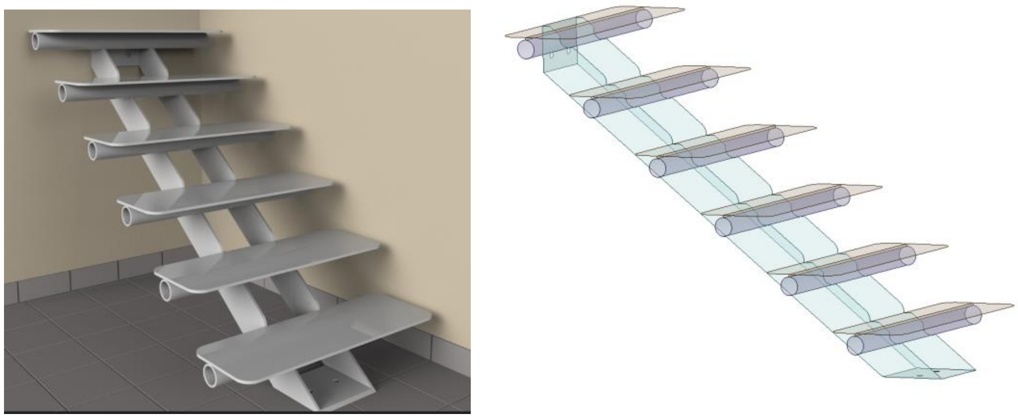

Figure 1: Belleville Washer

To determine if the design is acceptable or not, a Heel-Drop impact simulation was done (Figure 2). The conditions for an acceptable design are shown in Figure 4.

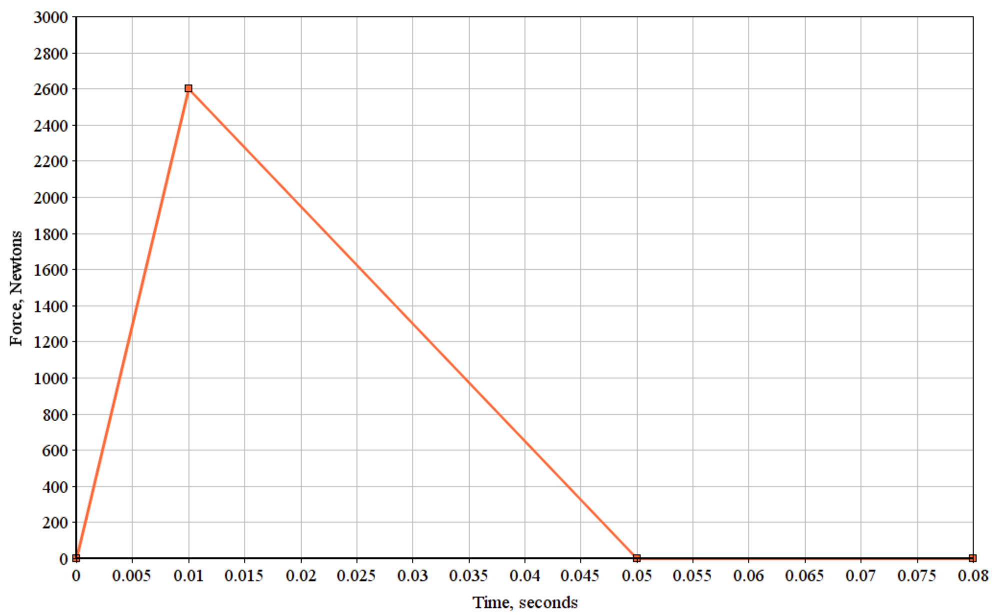

Figure 2: Heel-Drop Impact

1. Setup

The first step was to import the model to Ansys Mechanical in a Modal module and add the 25% of a maximum live-load mass of 200 kg. This mass was distributed on the top surface of the 6 steps as shown in Figure A.1. A default mesh was used for the analysis (shown in Figure A.2). The boundary conditions are shown in Figure A.3. The material used was Structural Steel.

2. Results and Analysis

The results for the Modal analysis are shown in Table 1.

Mode	Frequency [Hz]
1	28.96
2	38.235
3	43.854
4	57.785
5	83.616
6	88.438
7	103.25
8	109.81
9	118.84
10	133.08
11	134.15
12	139.26

Table 1: Modal analysis results

The first 3 modes represent the most excitable modes when a single force is applied in a specific direction. These 3 modes are shown in Figure A.4, Figure A.5 and Figure A.6. Depending on the position in which the heel is impacting the step, mode 2 or 3 will be excited. To simplify the analysis, the heel impact is assumed to be applied as a force distributed symmetrically on the surface of one step. This means that mode 3 should be excited. Now, because it is not completely clear in which step the force should be applied, 6 different cases were analyzed (1 for each step). For each case, a force of 1 N and phase angle of 0 were applied to the surface. These cases are shown in Figure A.7, Figure A.9, Figure A.11, Figure A.13, Figure A.15 and Figure A.17. For every case a 2.5% of critical damping at the dominating response frequency (43 Hz) was applied. The results are shown in Figure A.8, Figure A.10, Figure A.12, Figure A.14, Figure A.16 and Figure A.18. To check which case is the worst, Table 2 shows the max deformation obtained at the peak, close to 43.85 Hz.

Case	Amplitude (mm) at 43.85 Hz for a 1 N load
1	2e-5
2	3.77e-4
3	1.11e-3
4	1.61e-3
5	1.16e-3
6	3.46e-4

Table 2: Max amplitude obtained at 43.85 Hz for different cases.

It is clear from the table that the max deformation is obtained when the force is applied on the step 4. This is the case considered for the transient analysis when the heel hits the step. The settings for the transient analysis are explained below. The boundary conditions are shown in Figure A.18.

- **Time Step 1 (from 0 s to 0.02 s):** Auto Time Stepping On.
    - **Initial Substeps**: 20
    - **Minimum Substeps:** 20
    - **Maximum Substeps:** 1000

- **Time Step 2 (from 0.02 s to 0.05 s):** Auto Time Stepping On.
    - **Initial Substeps**: 20
    - **Minimum Substeps:** 20
    - **Maximum Substeps:** 1000

- **Time Step 1 (from 0.05 s to 0.08 s):** Auto Time Stepping On.
    - **Initial Substeps**: 20
    - **Minimum Substeps:** 20
    - **Maximum Substeps:** 1000

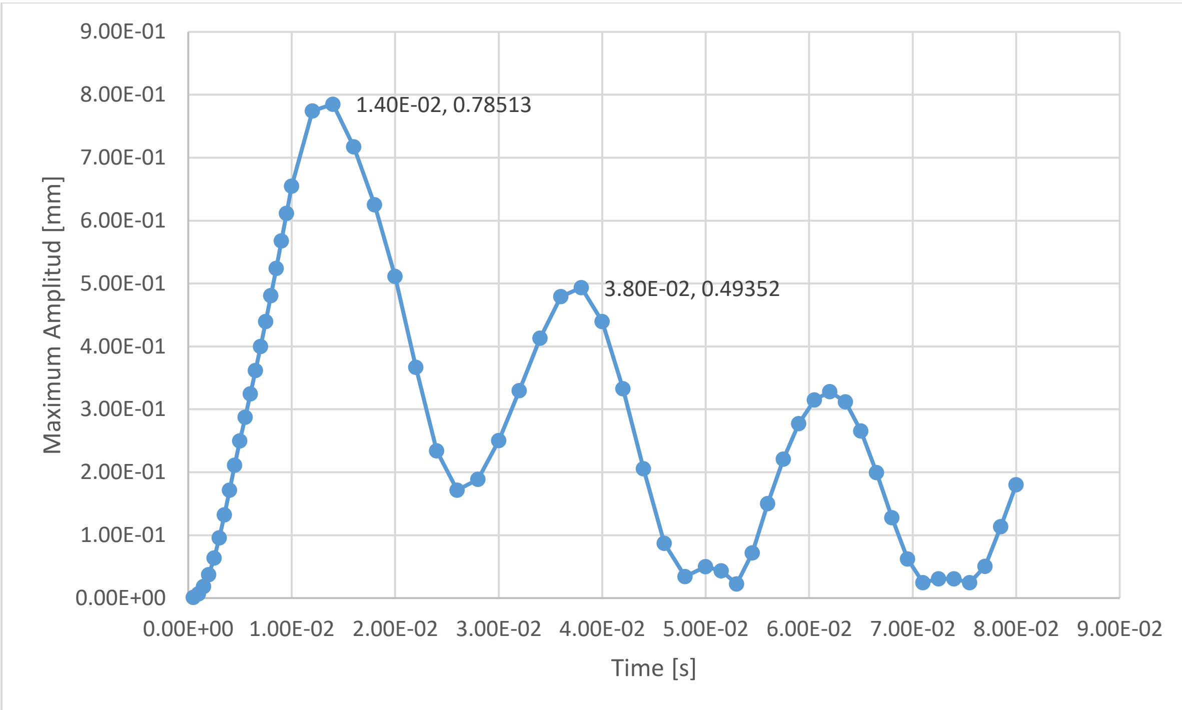

Figure 3: Maximum deformation for Transient Analysis

Figure 3 shows that the peak of deformation is reached when the heel hits the step at 0.01 s. The deformation at that moment is **0.78513 mm** and it is located at the edge of the step 4 as shown in Figure 5. From Figure 3, it can also be seen that the time between the peak at 0.014 s and the next peak at 0.038 s (when the step is freely vibrating) is **0.024 s**, which is the same as a frequency of **41.66 Hz**. This frequency represents the natural frequency of vibration and is similar to the one obtained for mode 3, meaning that mode 3 is being excited with the heel impact. Looking at the curve shown in Figure 4, for a frequency of 43 Hz and an amplitude 0.785 mm, the values are located in the zone where the vibration is strongly perceptible, meaning that the design is **not acceptable**.

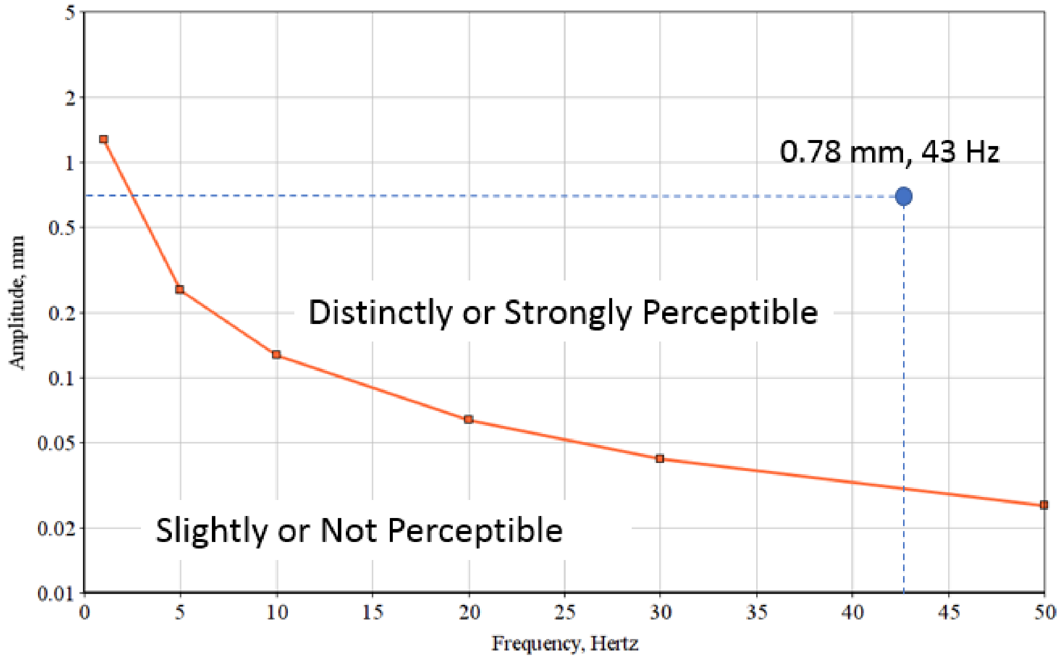

Figure 4: Maximum deformation for Transient Analysis

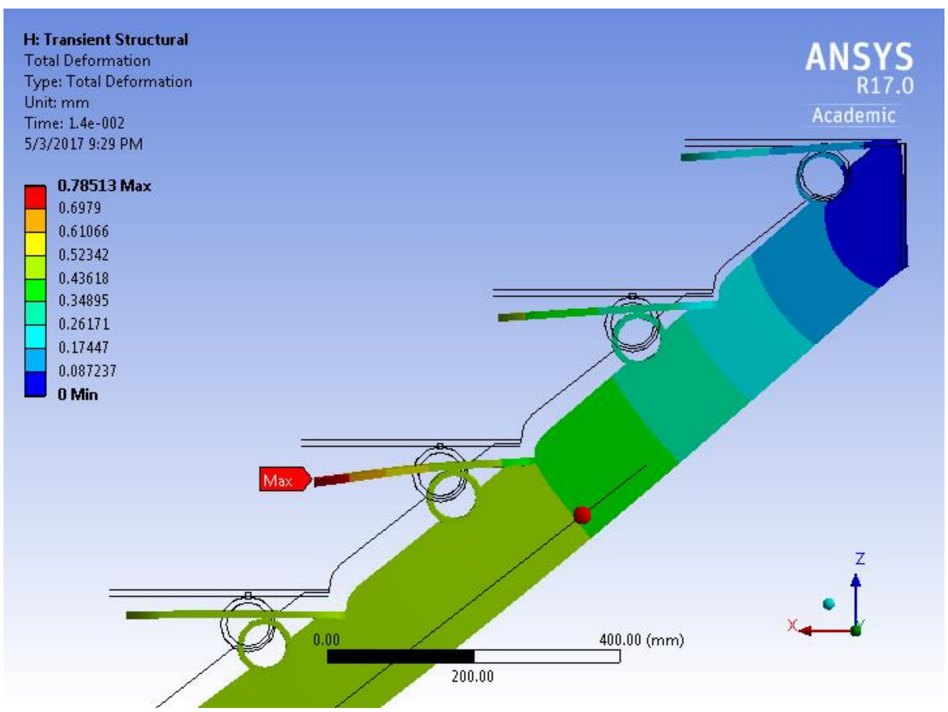

Figure 5: Maximum deformation for Transient analysis at t=0.014 s

3. Appendix

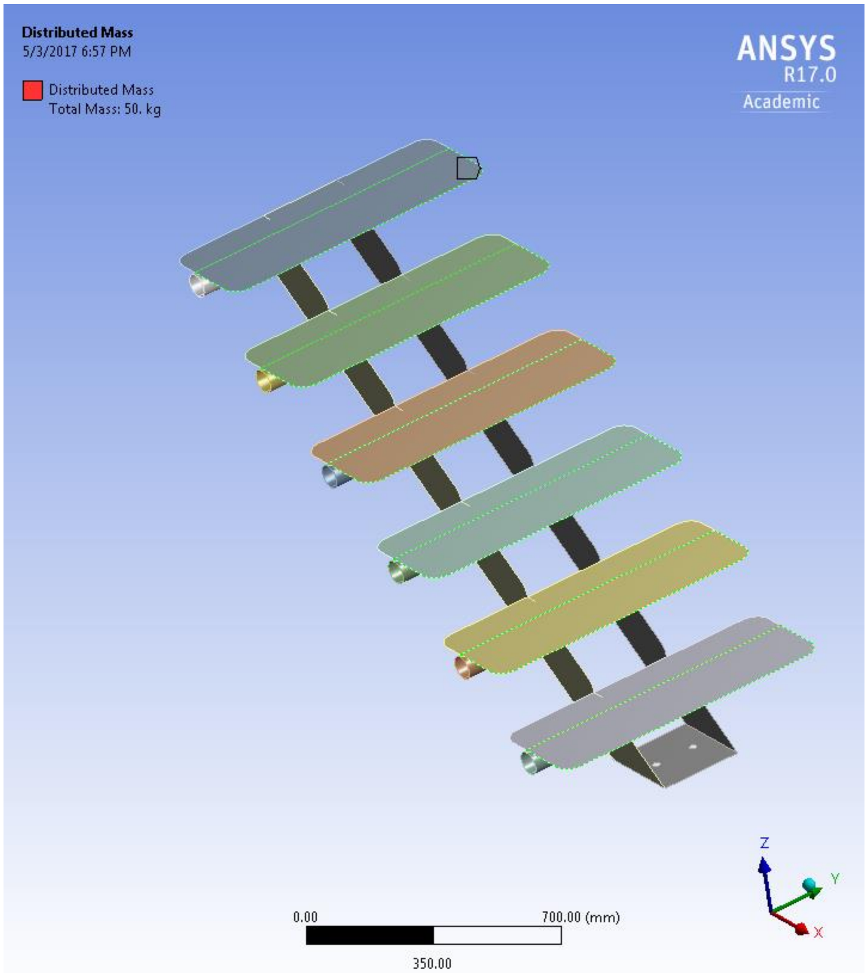

Figure A.1: Distributed Mass

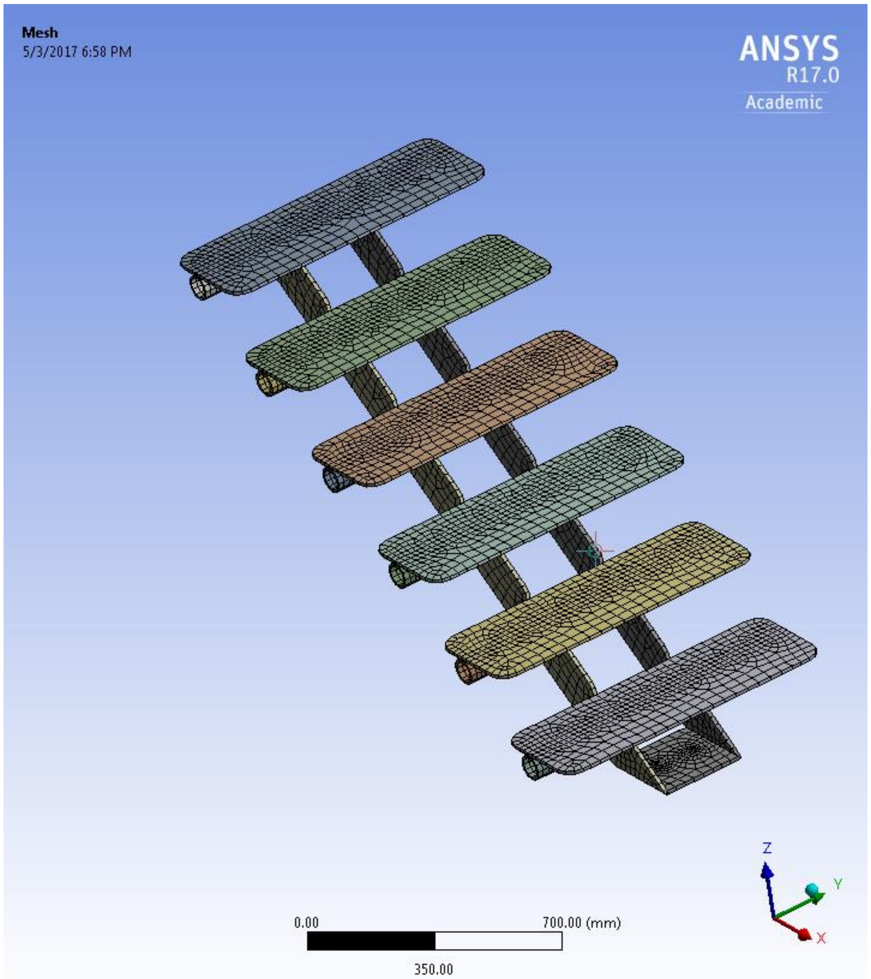

Figure A.2: Default Mesh

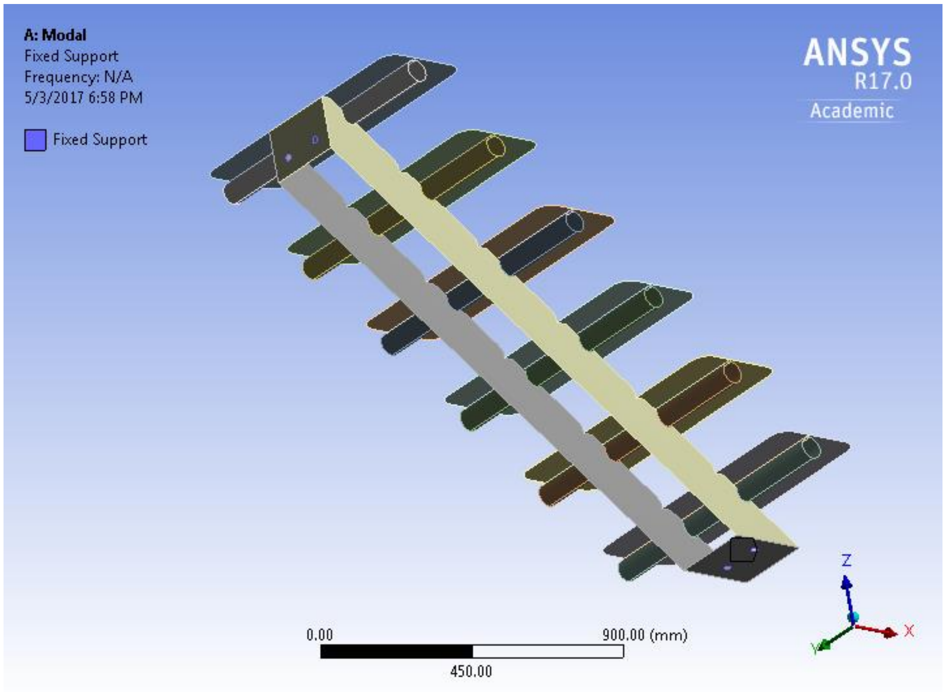

Figure A.3: Boundary conditions for Modal analysis

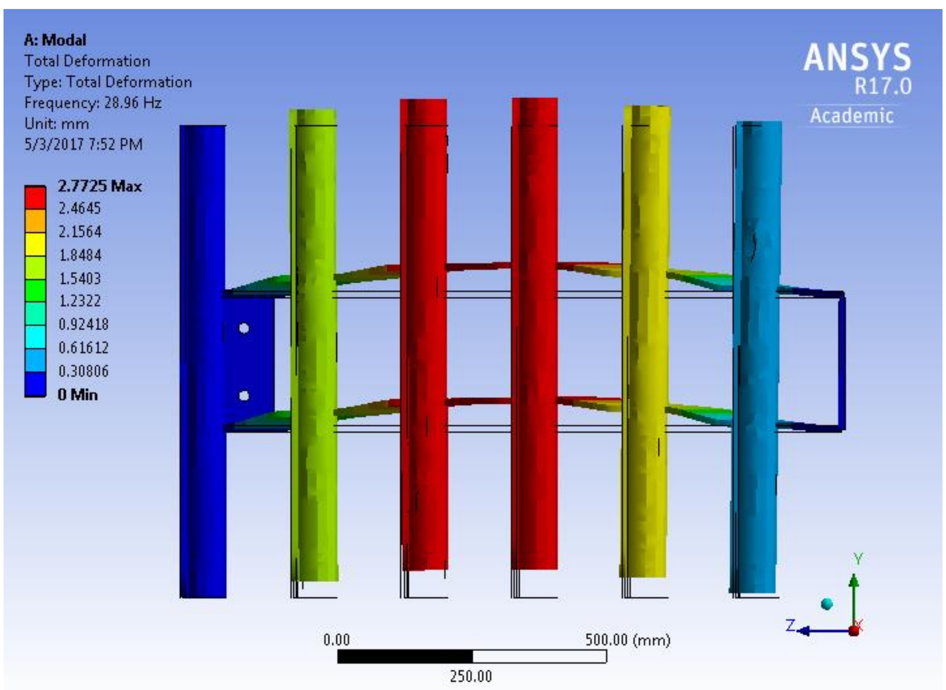

Figure A.4: Mode 1 (28.96 Hz)

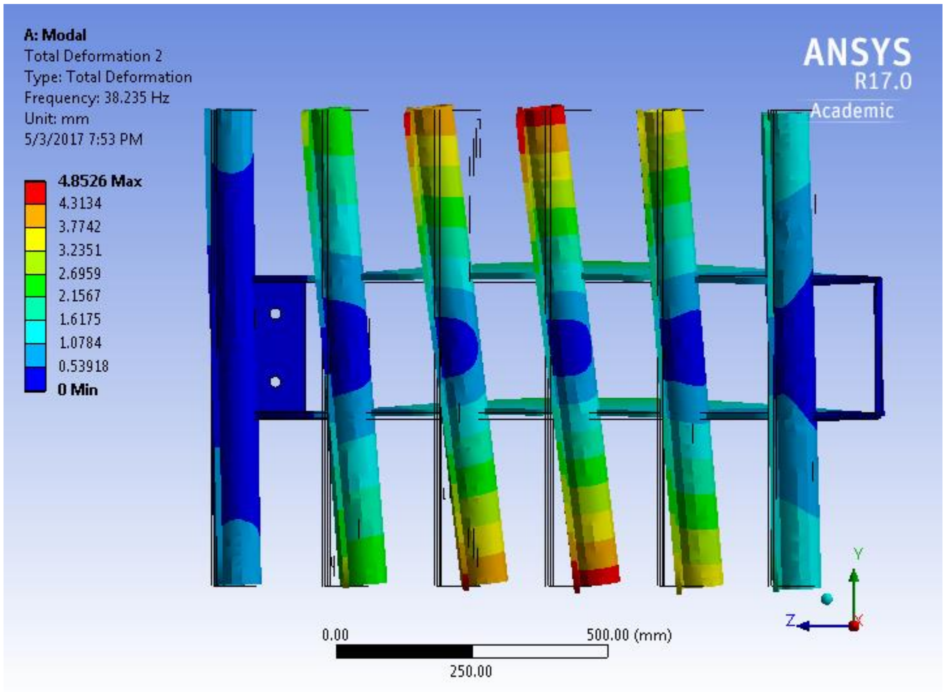

Figure A.5: Mode 2 (38.235 Hz)

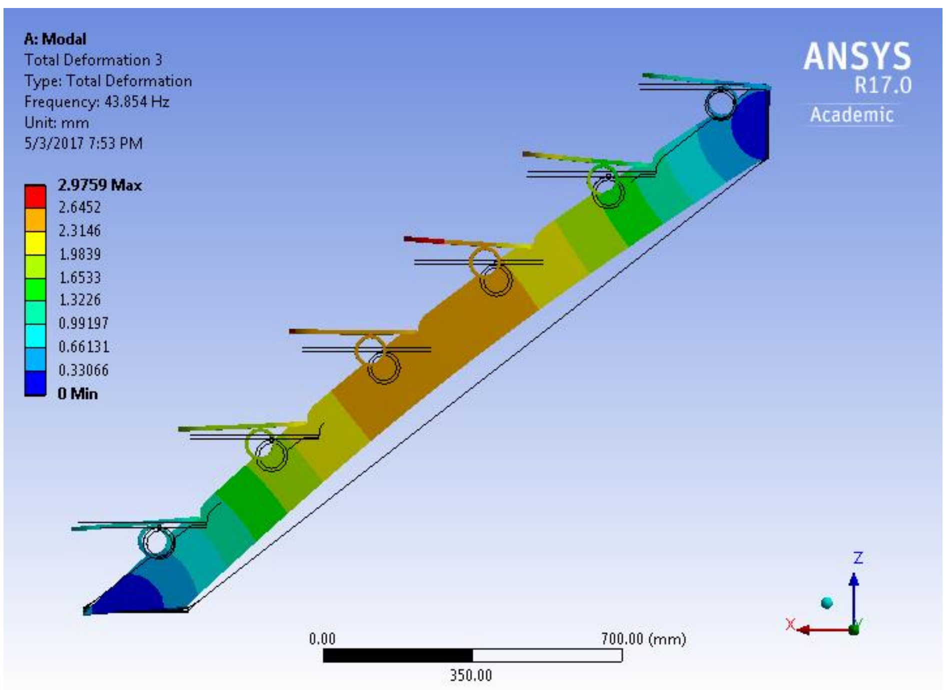

Figure A.6: Mode 3 (43.85 Hz)

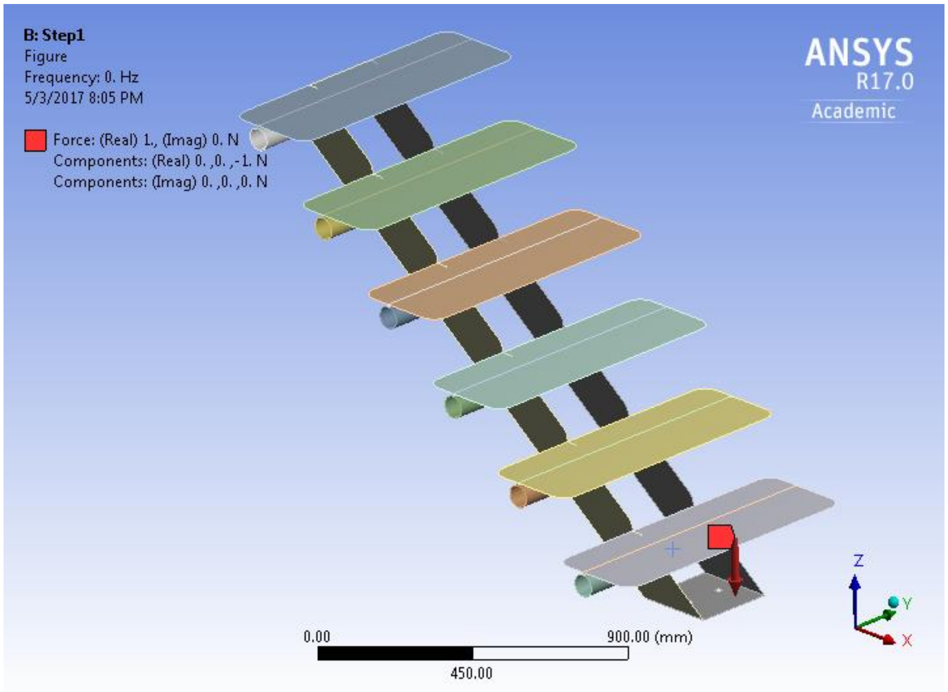

Figure A.7: Harmonic Response, Case 1

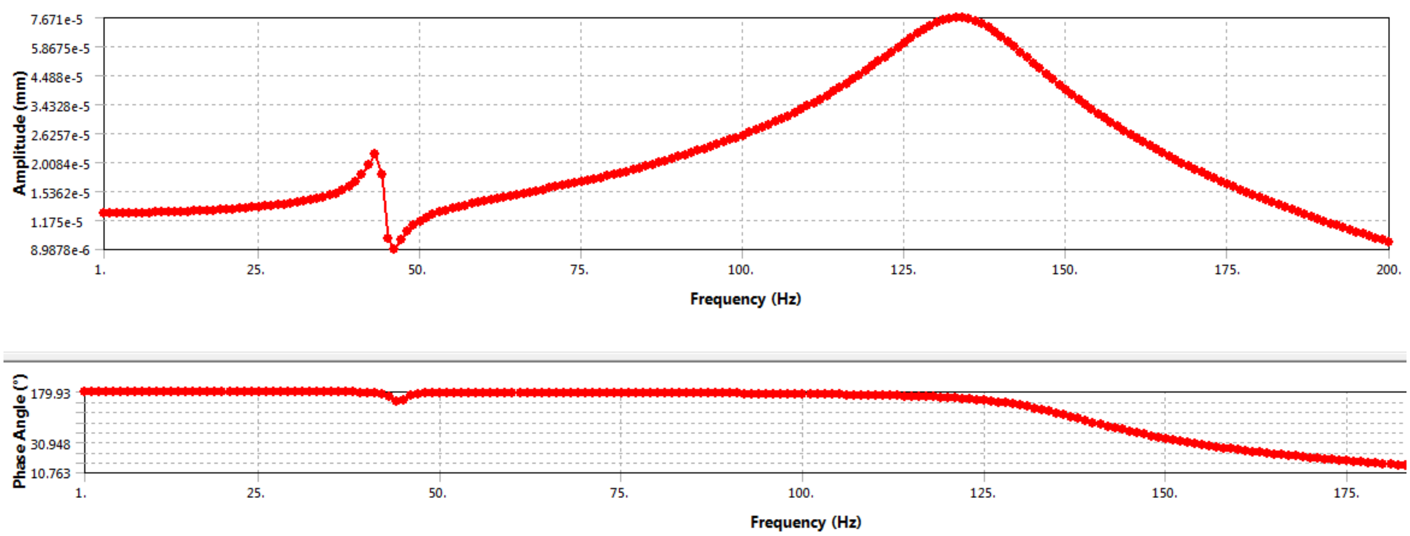

Figure A.8: Frequency Response, Max Deformation, Case 1

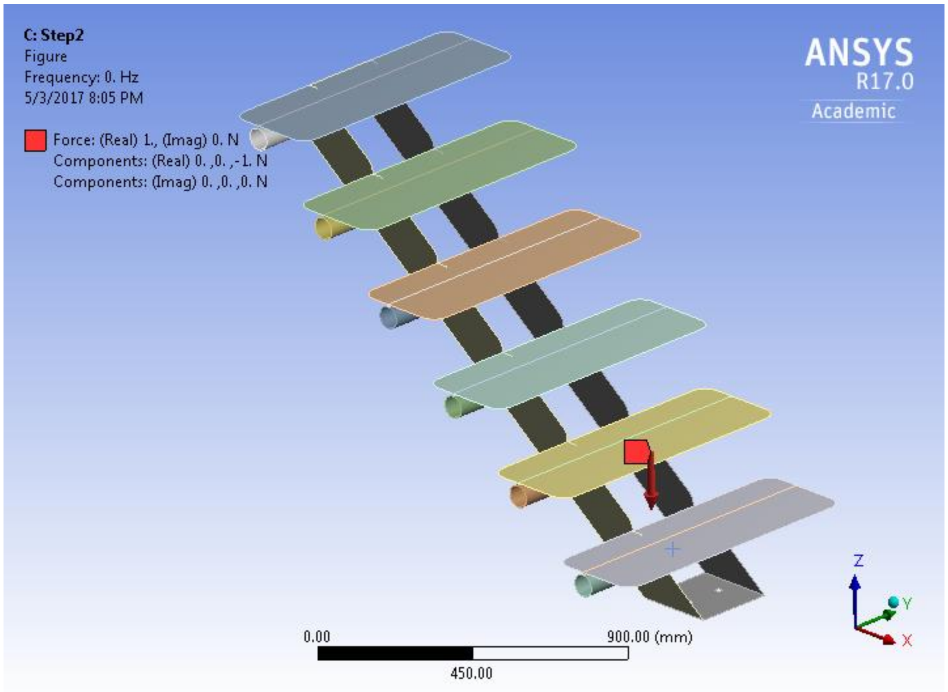

Figure A.9: Harmonic Response, Case 2

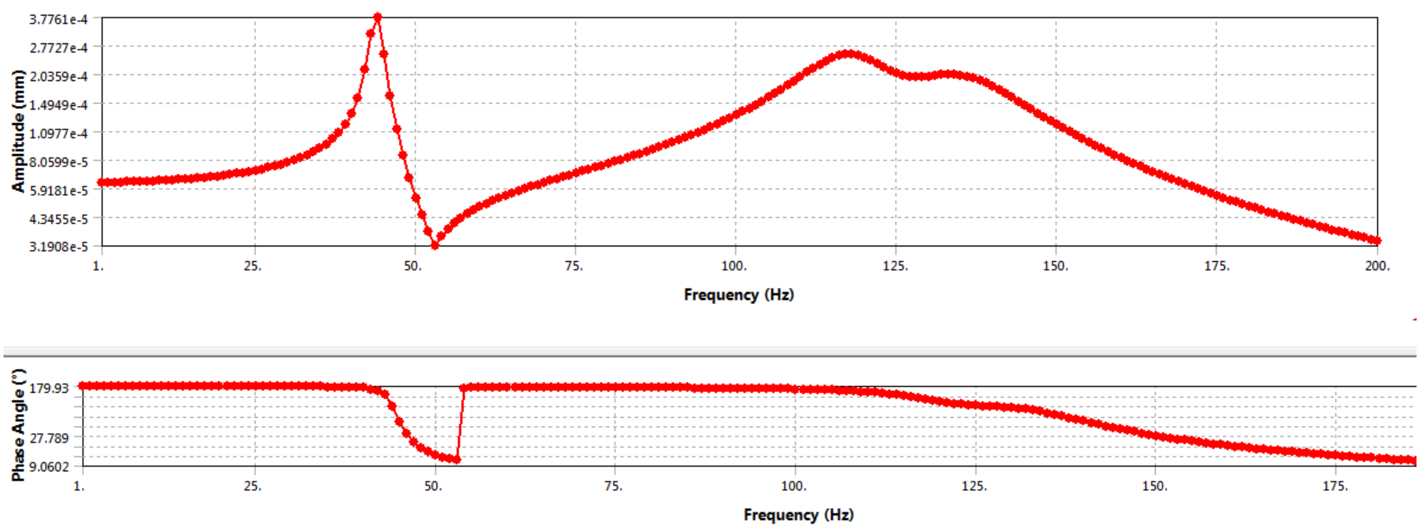

Figure A.10: Frequency Response, Max Deformation, Case 2

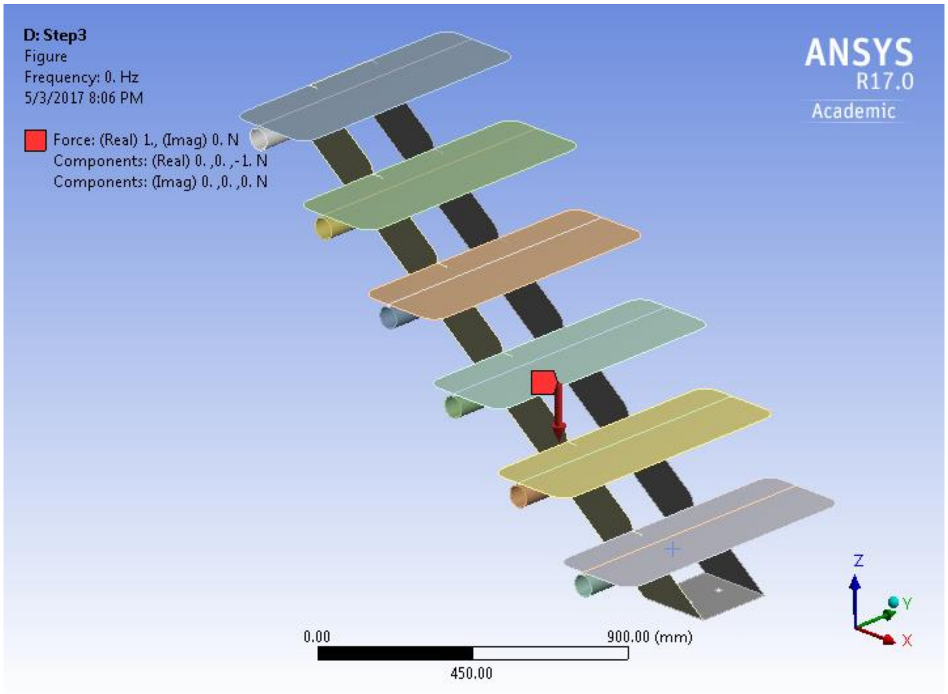

Figure A.11: Harmonic Response, Case 3

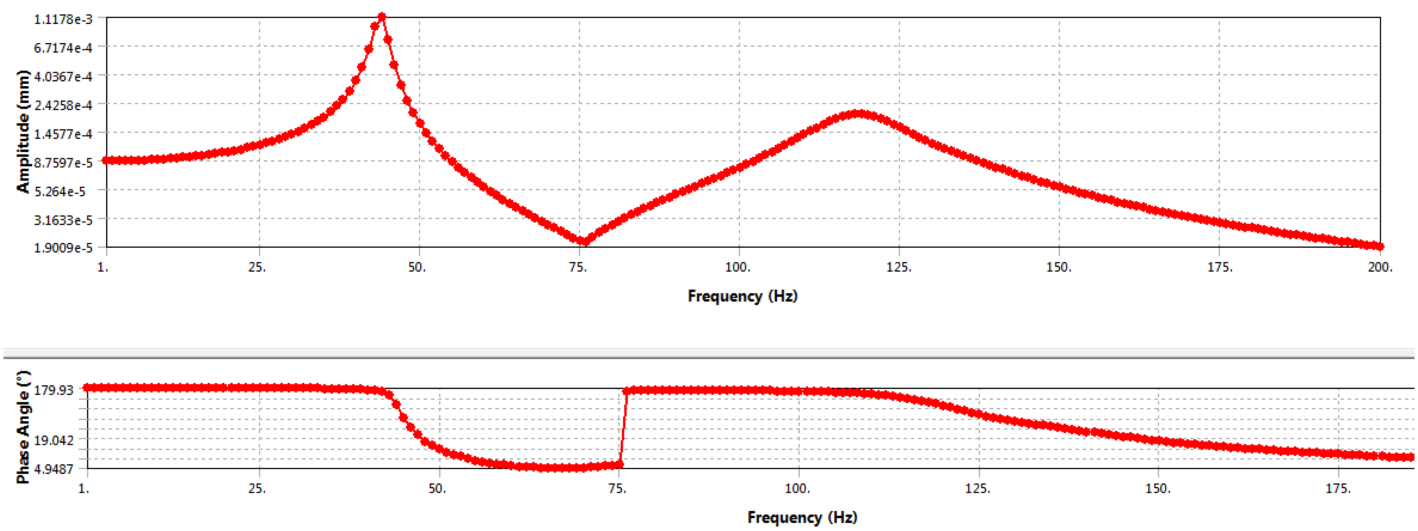

Figure A.12: Frequency Response, Max Deformation, Case 3

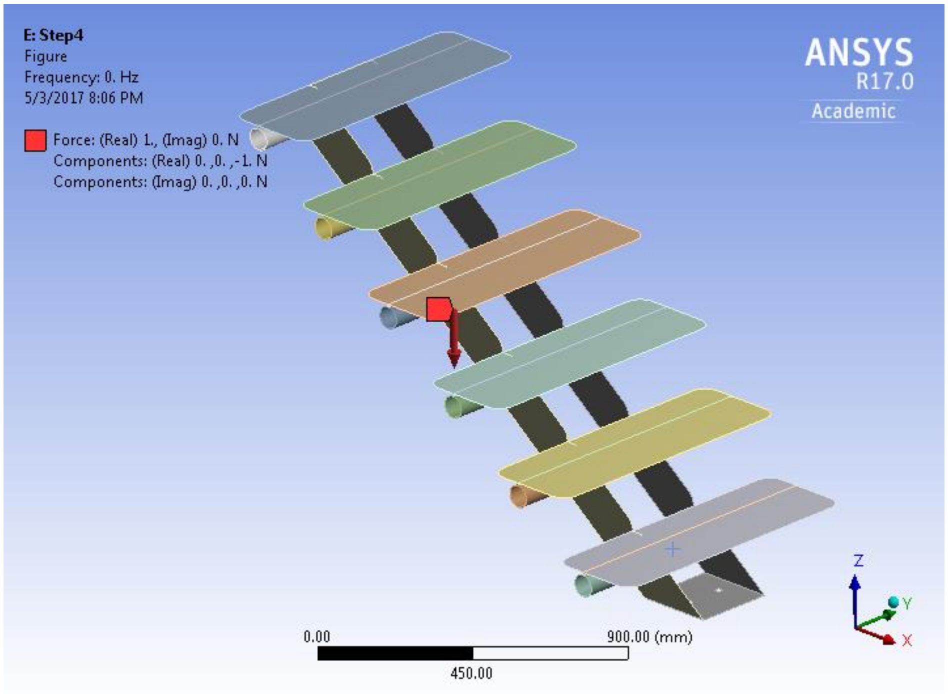

Figure A.13: Harmonic Response, Case 4

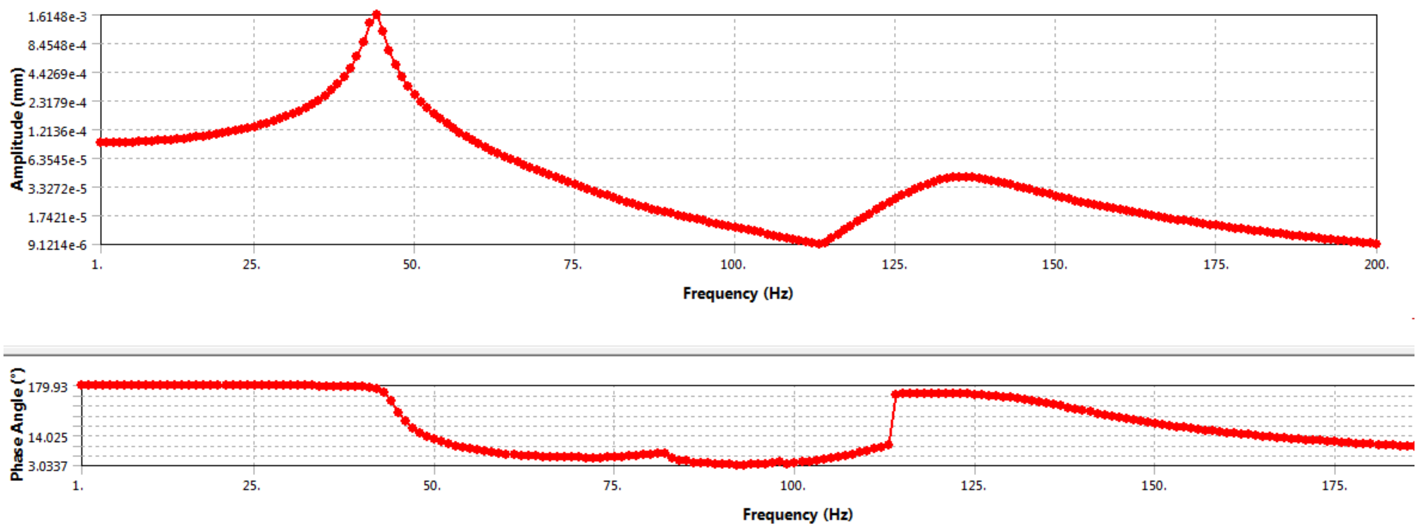

Figure A.14: Frequency Response, Max Deformation, Case 4

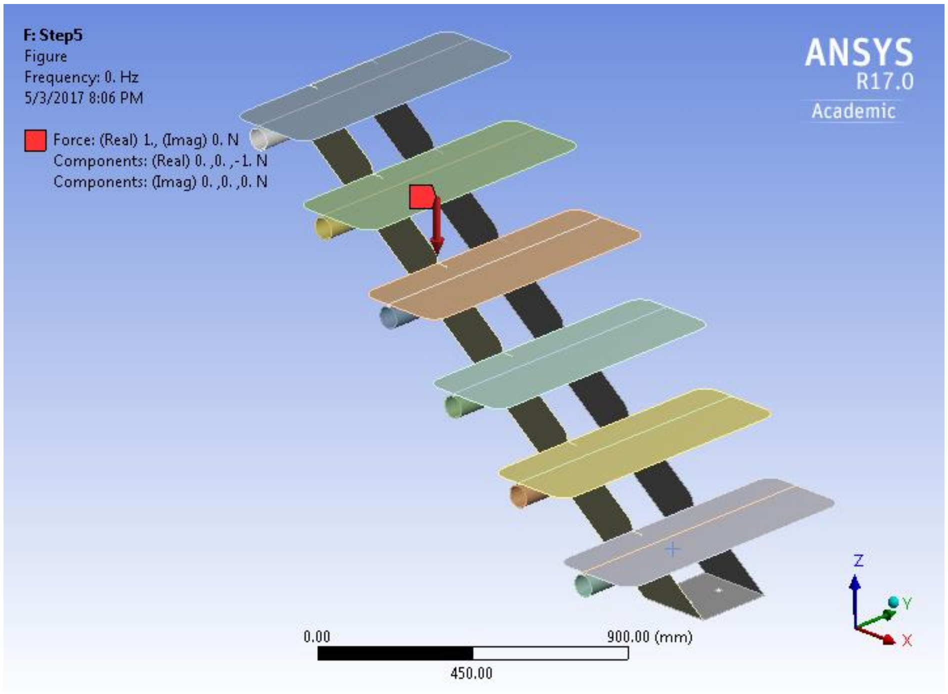

Figure A.15: Harmonic Response, Case 5

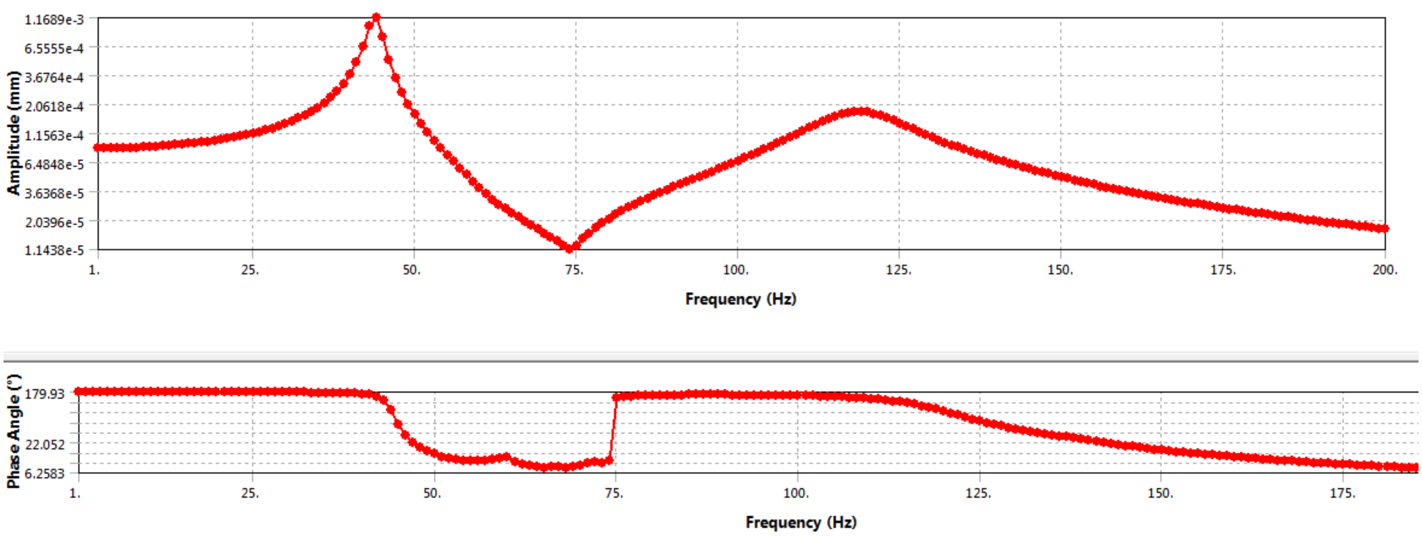

Figure A.16: Frequency Response, Max Deformation, Case 5

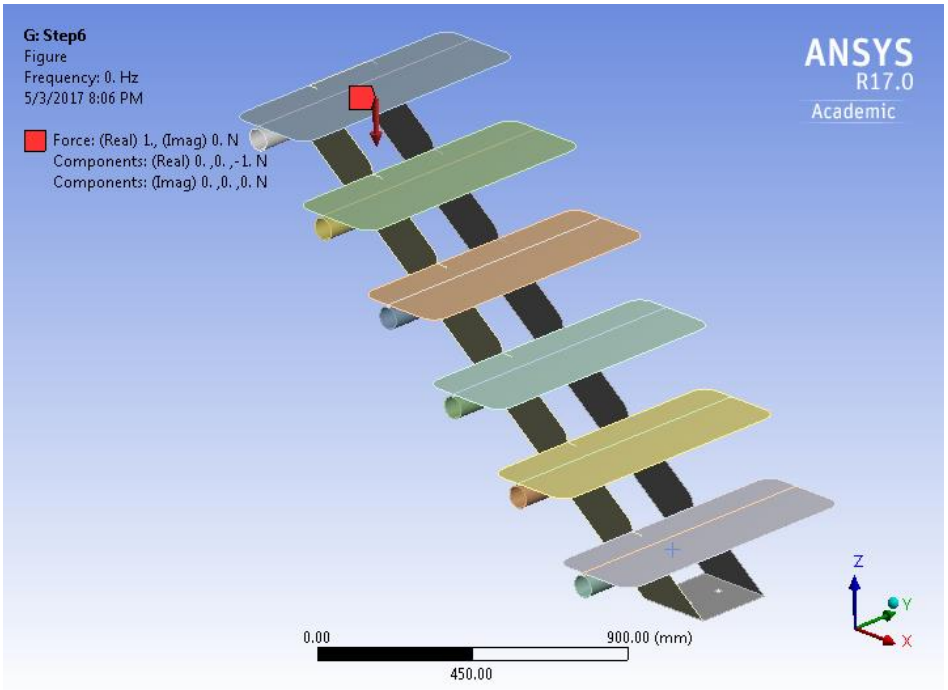

Figure A.17: Harmonic Response, Case 6

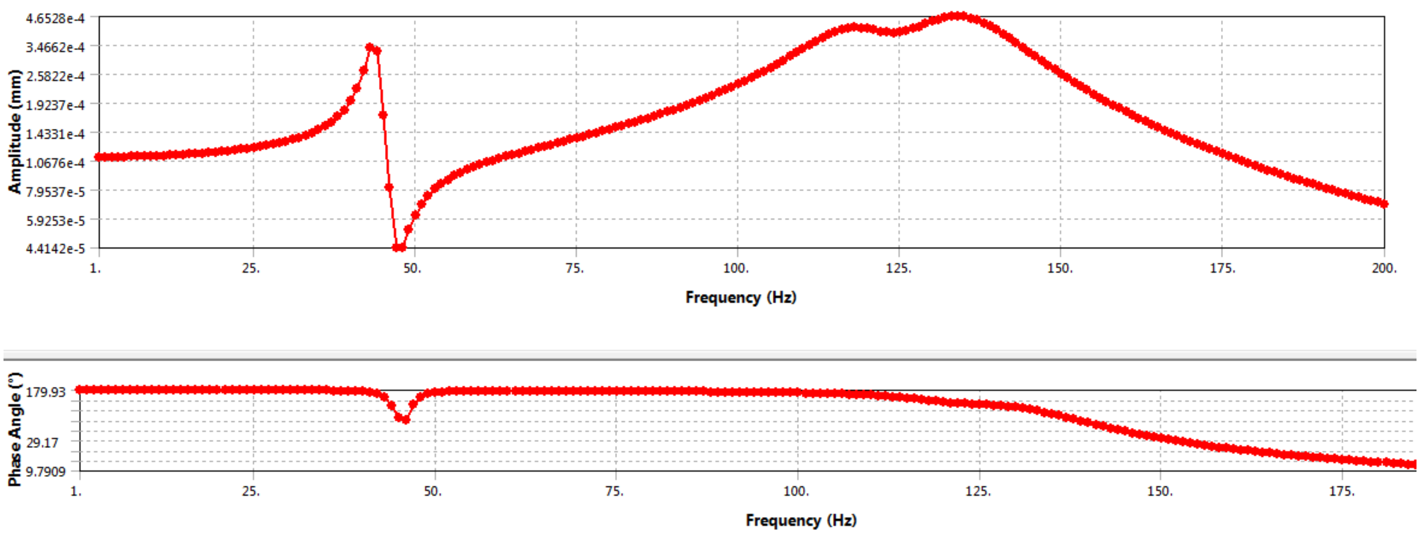

Figure A.18: Frequency Response, Max Deformation, Case 6

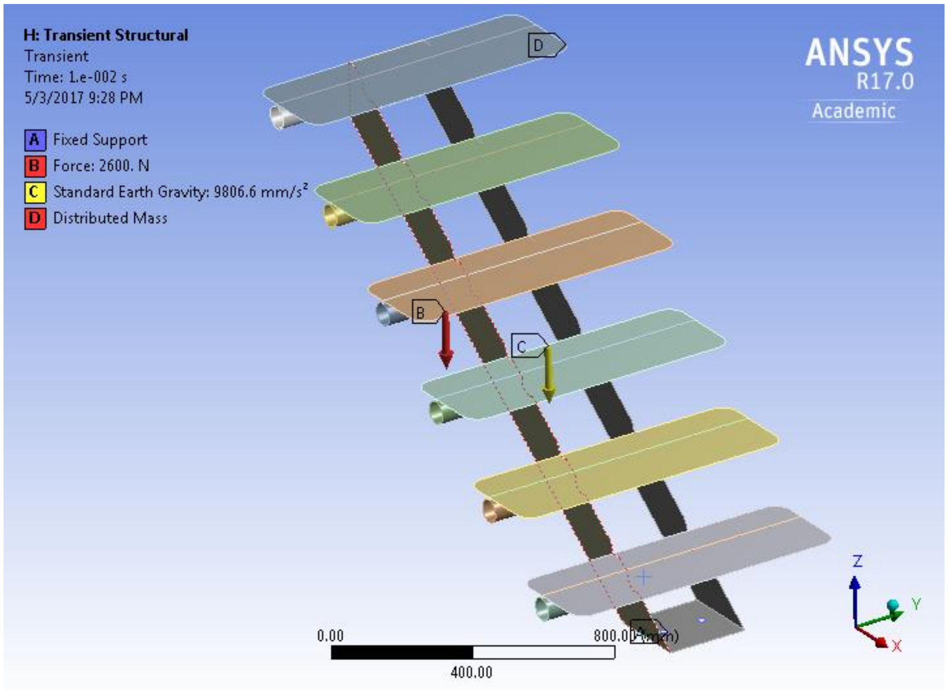

Figure A.19: Boundary conditions for Transient Structural Analysis.