

24-650 Applied Finite Element Analysis

Homework No 3

Modal analysis of a tuning fork

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The objective of this assignment was to perform a modal analysis of a Concert “A” tuning fork (440 Hz) to verify the tuning fork frequency and then modify the design provided. A tuning fork is an acoustic resonator in the form of a two-pronged fork with the prongs formed by a U-shaped bar of elastic metal (usually steel). It resonates at a specific constant pitch [1]. The dimensions are shown in *Figure 1*.

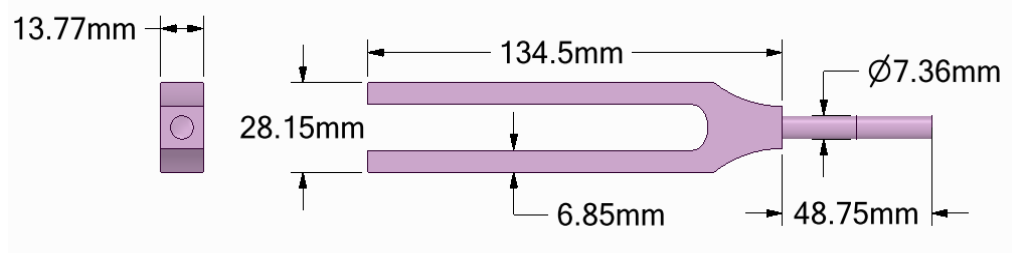


Figure 1: Tuning fork dimensions

1. Setup

The first step was to create a new project in Ansys Workbench and to set a Modal Analysis module. Using the default Engineering Data option, I started setting the conditions for the analysis.

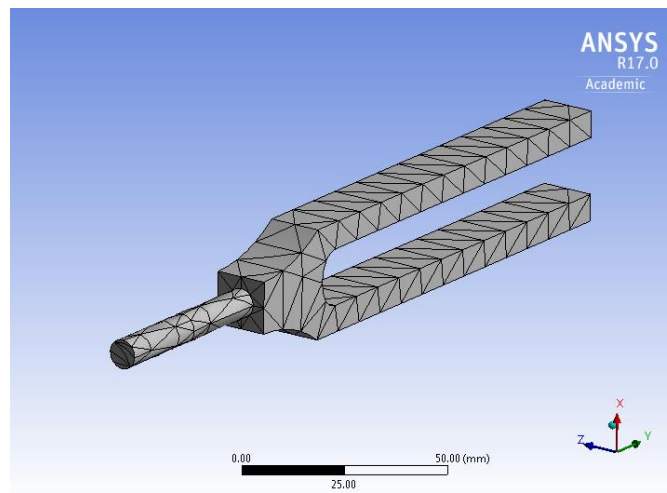


Figure 2: Default Mesh

The **default mesh** consisted of **1,044 nodes** and **455 elements**. This can be seen in *Figure 2*

With the mesh ready, I applied a **fixed support** to **surface “B”** shown in *Figure A.1*, and then proceeded to do the modal analysis for **structural steel** and **10 max modes to find**. The values obtained for frequencies are shown in *Table 1* and the Modes are shown in *Figure A.3* through *Figure A.12*

2. Results and Analysis

Mode	1	2	3	4	5	6	7	8	9	10
Frequency [Hz]	131.43	136.83	448.5	660.42	1150.2	1598.5	1951.1	2746.3	3592.6	5102.6

Table 1: Modes Frequency for default mesh and surface B fixed (Case 1)

As can be seen, **Mode 3**, called Fundamental Mode, has a frequency of **448.5 Hz**, which is close to the 440 Hz that correspond to note A. The difference in value for the frequency is due to the size of the mesh elements. By refining the mesh, we could get a better result. To understand why the fork doesn't produce sound at the lower modes, it is first necessary to understand why a tuning fork produces sound. As shown in *Figure 3*, the oscillation of the prongs makes the particles of air immediately in front of them transmit progressively in the form of a compression wave (when the prongs move outwards) and in the form of a rarefaction wave (when the prongs move inwards), creating a sound wave.

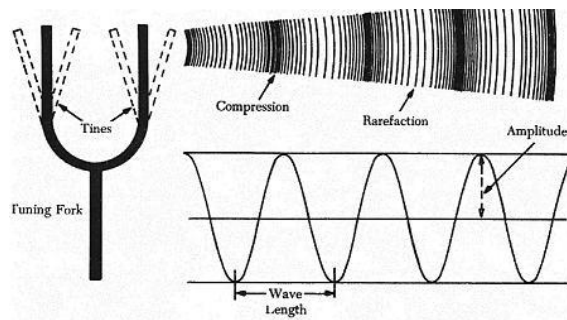


Figure 3: Sound propagation by a tuning fork [2]

According to *Figure 3*, most of the modes should produce a sound wave that we could hear. This is true, but we don't hear other frequencies because of the damping effect that our hand produces while holding the tuning fork. As is shown in *Figure 4*, there are **symmetric modes** and **antisymmetric modes**. For the tuning fork, there are 2 symmetric modes (Mode 3 and Mode 8) and 8 antisymmetric modes (Mode 1, 2, 4, 5, 6, 7, 9, 10). For the symmetric modes the two prongs move equally but in opposite directions. This creates a very small transmission effect from the prongs to the base rod, so by holding the tuning fork with your hand there is almost no effect on the vibration of the prongs, and the sound wave is created. Evidence of this can be seen in the modal stresses in *Figure A.13* and *Figure A.14*, where it is important to note that there are no stresses on the handle rod. Conversely, an antisymmetric mode makes the prongs move in any direction and transmit movement from the prongs to the base rod, so if you hold the tuning fork by the handle, you will be damping the vibration of those modes.

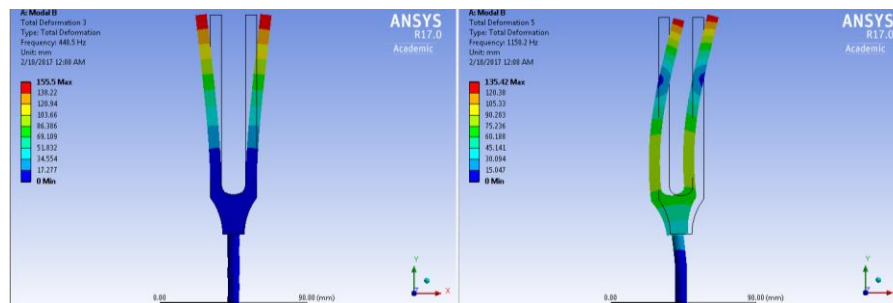


Figure 4: Symmetric mode vs Antisymmetric mode

Per the above explanation, symmetric modes can produce sound, so there also should be a heard frequency of **2,746 Hz** with **Mode 8** (*Figure A.10*). This mode is called Clang Mode and it results from striking the prongs of the fork against a hard object. The frequency of the Clang Mode is around 6.26 times higher than the fundamental mode [3].

If we change the **fixed support** to **surface “A”** or **“C”** (shown in *Figure A.15* and *Figure A.16*) we still get the same frequencies for the Fundamental Mode and for the Clang Mode (shown in *Table 2*, *Figure A.17* and *Figure A.18*). The reason for this is the same as previously explained. The hand holding the rod doesn't produce any effect on the vibration for the symmetric modes, so the frequencies of vibration aren't affected by changing the fixed support.

Fixed Support	Fundamental Mode Frequency [Hz]	Clang Mode Frequency [Hz]
Surface B	448.5	2,746.3
Surface A	448.5	2,746.8
Surface C	448.5	2,745.4

Table 2: Modes for default mesh and surface B fixed

To perform a mesh sensitivity study, I used the **convergence object** with **4 Max Refinement Loops** and a value of **2 for Refinement Depth**. The **Allowable Change** for the Frequency of Mode 3 was defined as **1%**. With this, the new refined mesh consisted of **44,000 nodes** and **28,497 elements**. This is shown in *Figure A.19*. The Convergence plot and Mode frequency for Case 1 are shown in *Figure A.20* and *Figure A.21*. In the table below, values are compared for the default mesh and the refined mesh.

Mode	1	2	3	4	5	6	7	8	9	10
Frequency [Hz] (Case1, Default Mesh)	131.43	136.83	448.5	660.42	1150.2	1598.5	1951.1	2746.3	3592.6	5102.6
Frequency [Hz] (Case1, Refined Mesh)	126.93	130.79	440.37	643.33	1126.9	1548.6	1850.9	2699.3	3398.6	4787.5

Table 3: Mode Frequency for default mesh and refined mesh with surface B fixed (Case 1)

As illustrated in *Table 3*, the new frequency obtained for Mode 3 is **440.37 Hz**, which differs by less than 1% of 440 Hz, which is the frequency of the note A.

The next step was to change the material to **Aluminum** and see how the frequencies change. This is shown in *Figure A.22*. In the table below, the values for the steel and the aluminum are compared.

Mode	1	2	3	4	5	6	7	8	9	10
Frequency [Hz] (Steel)	131.43	136.83	448.5	660.42	1150.2	1598.5	1951.1	2746.3	3592.6	5102.6
Frequency [Hz] (Aluminum)	132.33	137.89	450.46	659.33	1155.1	1592.7	1960	2758	3611.6	5089.5

Table 4: Mode Frequency for steel and aluminum with surface B fixed and default mesh (Case 1)

If we consider the tuning fork a simple spring-mass system with stiffness of k and a mass of m , the equation and analytical solution are shown below:

$$m\ddot{x} + kx = 0$$

$$x(t) = A \cos(2\pi f_n t)$$

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Where, **A** is the amplitude of oscillation and **f_n** is the **undamped natural frequency**. Now, considering that the prongs are two cantilever beams, the stiffness of which depends on the material and cross section geometry, we can define the undamped natural frequency for the tuning fork as:

$$f_n = \frac{1.875^2}{2\pi l^2} \sqrt{\frac{EI}{\rho A}}$$

Where, 1.875 is the smallest positive solution for $\cos x \cosh x = -1$, l is the length of the prongs in meters, E is the Young's modulus of the material in pascals, I is the second moment of area of the cross-section in meters to the fourth power, ρ is density of the material in kilograms per cubic meters and A is the cross-sectional area of the prongs in square meters [1]. Using the analytic expression, we can obtain the values for frequency and compare them to the values obtained in Ansys. This is shown in *Table 5*.

	Length of prongs [m]	Young's Modulus [Pa]	Second Moment of Area [m ⁴]	Density of the material [kg/m ³]	Cross-sectional area [m ²]	Natural frequency [Hz]
Steel	0.1125	2x10 ¹¹	3.68 x10 ⁻¹⁰	7,850	9.43 x10 ⁻⁵	441.26
Aluminum	0.1125	7.1x10 ¹⁰	3.68 x10 ⁻¹⁰	2,770	9.43 x10 ⁻⁵	442.59

Table 5: Natural Frequency for steel and aluminum using analytical expression.

As can be seen, the result changes by less than **0.5%** for both analytical value and simulation value, which is a negligible difference. The explanation of this is that by changing the material to aluminum, both the Young's Modulus and the density of the material decrease, but the value of E/ρ remains almost constant. This doesn't happen with **Copper**, with which the natural frequency obtained is about **318.25 Hz**.

In order to design a new fork for a specific note frequency, we need to check the analytical expression. If we want a frequency of **120 Hz**, we can change the material and geometry. Two different designs are shown below with the analytical values, and the simulation results are shown in *Figure A.23* through *Figure A.30*. For the Design 2, the material selected is Copper and the length of the prongs is 0.183 m. The cross-sectional area remains the same. For the Design 3, the material selected is Aluminum and the length of the prongs is 0.216 m. The cross-sectional area of the prongs changes from 9.43 x10⁻⁵ m² to 3.4 x10⁻⁴ m².

	Length of prongs [m]	Young's Modulus [Pa]	Second Moment of Area [m ⁴]	Density of the material [kg/m ³]	Cross-sectional area [m ²]	Natural frequency [Hz]
Design 2- Copper	0.183	1.1x10 ¹¹	3.68 x10 ⁻¹⁰	8,300	9.43 x10 ⁻⁵	120.27
Design 3- Aluminum	0.216	7.1x10 ¹⁰	1.33 x10 ⁻⁹	2,770	3.4 x10 ⁻⁴	120.06

Table 6: Natural Frequency for 2 different designs using analytical expression.

3. References

- [1] "Wikipedia," [Online]. Available: https://en.wikipedia.org/wiki/Tuning_fork.
- [2] [Online]. Available: <http://avstop.com/ac/apgeneral/sound.html>.
- [3] D. A. Russell, "Acoustics and Vibration Animations," [Online]. Available: <http://www.acs.psu.edu/drussell/Demos/TuningFork/fork-modes.html>.

4. Appendix

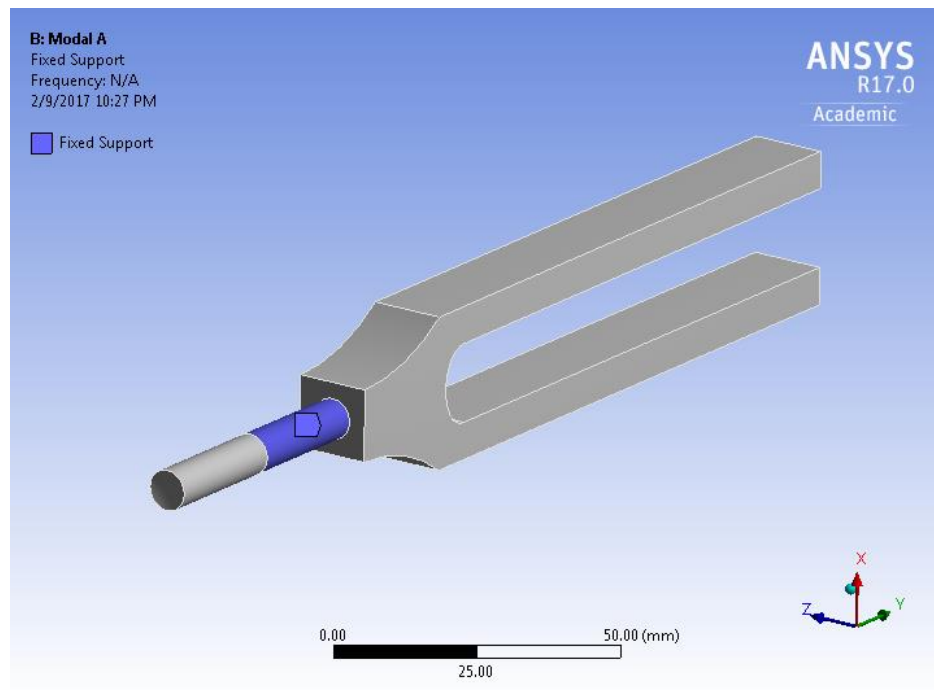


Figure A.1: BC, Fixed support, surface B (Case 1)

Tabular Data		
	Mode	<input checked="" type="checkbox"/> Frequency [Hz]
1	1.	131.43
2	2.	136.83
3	3.	448.5
4	4.	660.42
5	5.	1150.2
6	6.	1598.5
7	7.	1951.1
8	8.	2746.3
9	9.	3592.6
10	10.	5102.6

Figure A.2: Modes Frequency for Case 1 (default mesh)

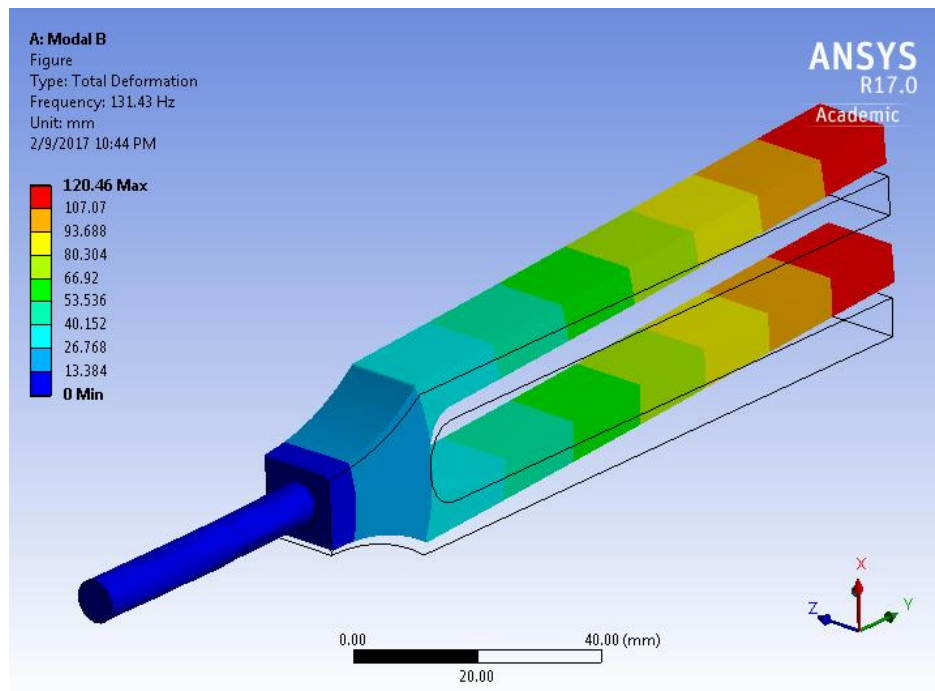


Figure A.3: Mode 1 for Case 1 (default mesh)

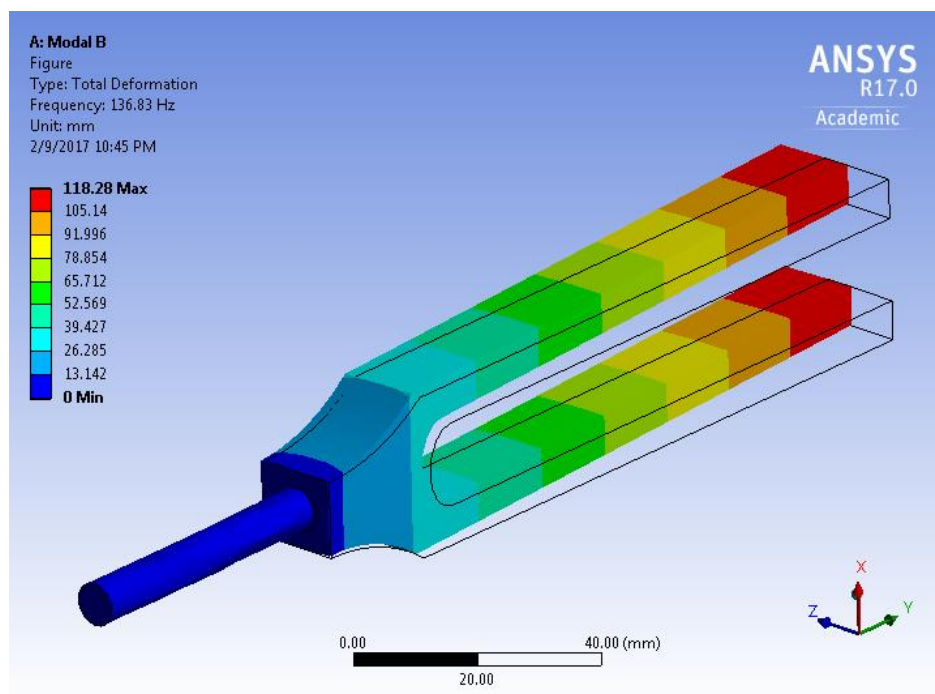


Figure A.4: Mode 2 for Case 1 (default mesh)

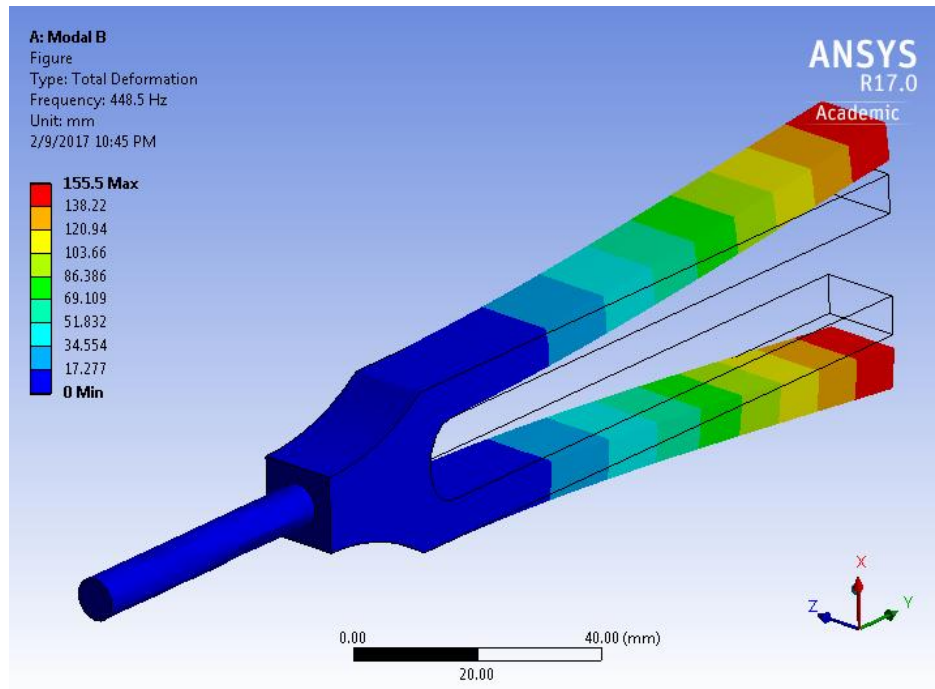


Figure A.5: Mode 3 for Case 1 (default mesh)

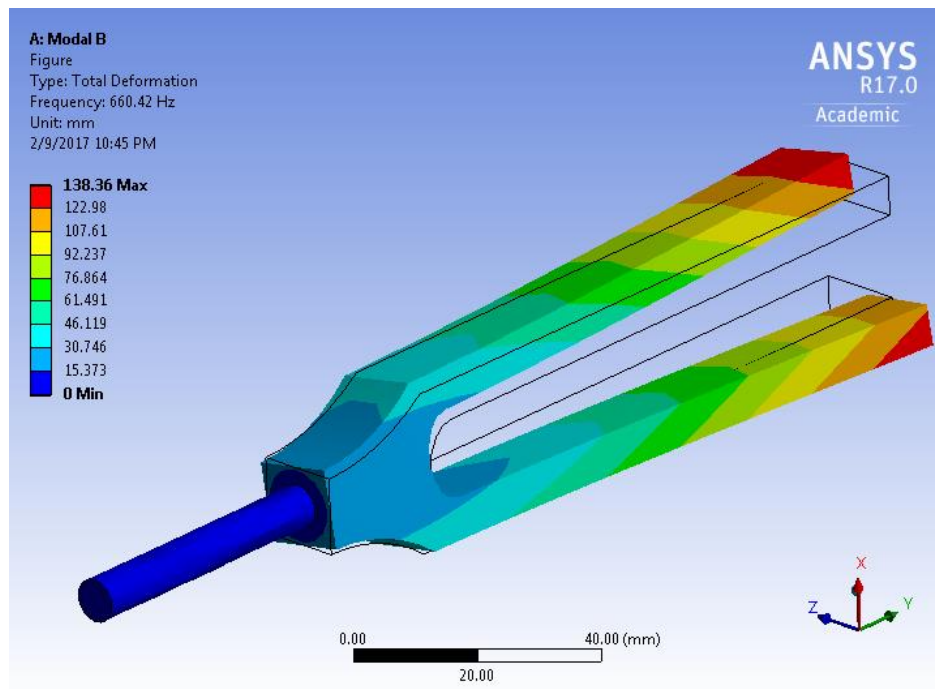


Figure A.6: Mode 4 for Case 1 (default mesh)

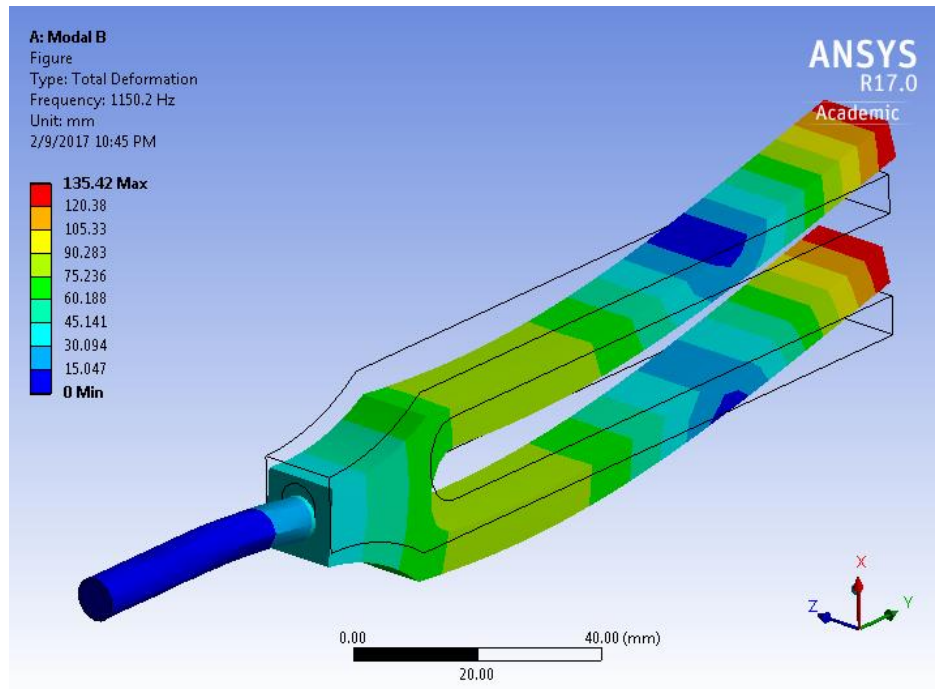


Figure A.7: Mode 5 for Case 1 (default mesh)

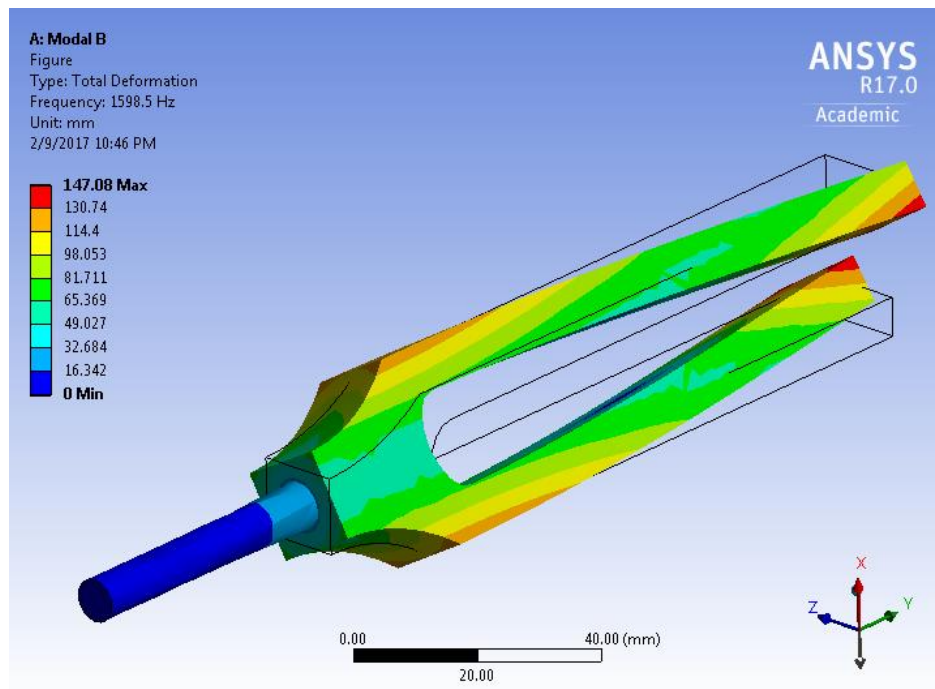


Figure A.8: Mode 6 for Case 1 (default mesh)

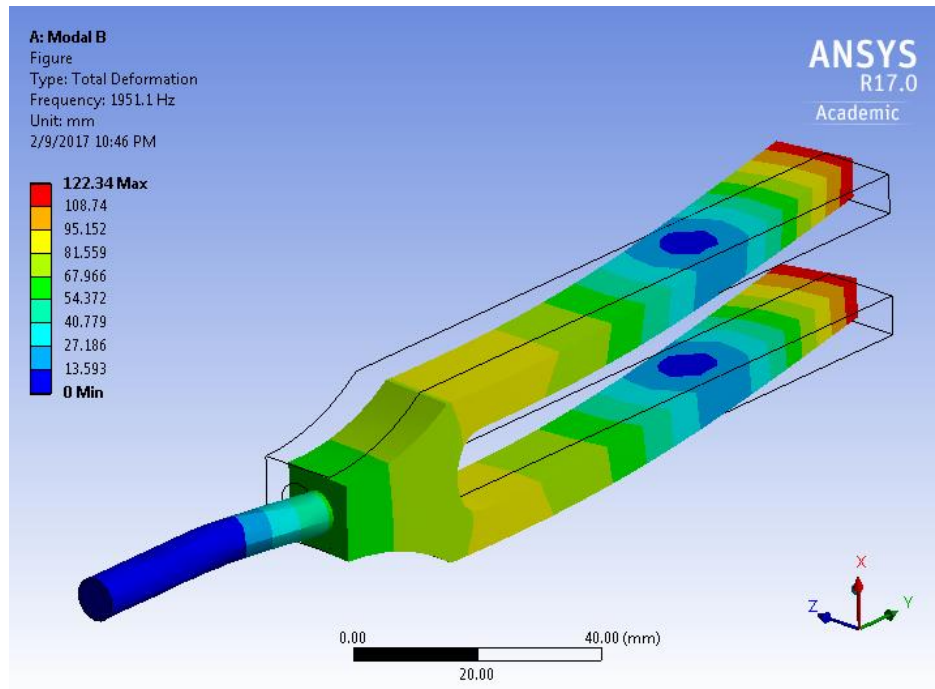


Figure A.9: Mode 7 for Case 1 (default mesh)

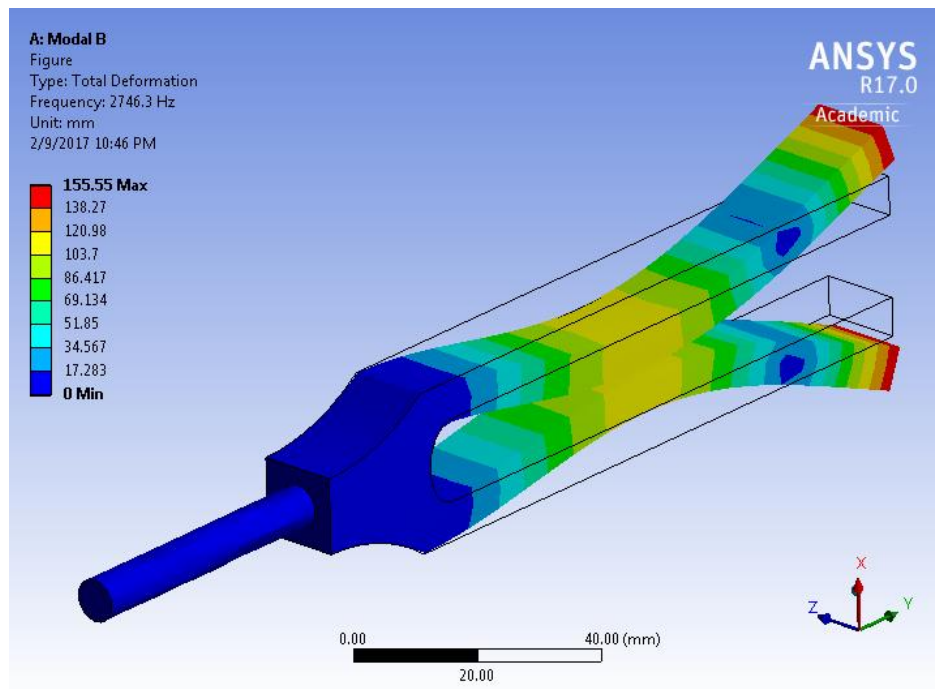


Figure A.10: Mode 8 for Case 1 (default mesh)

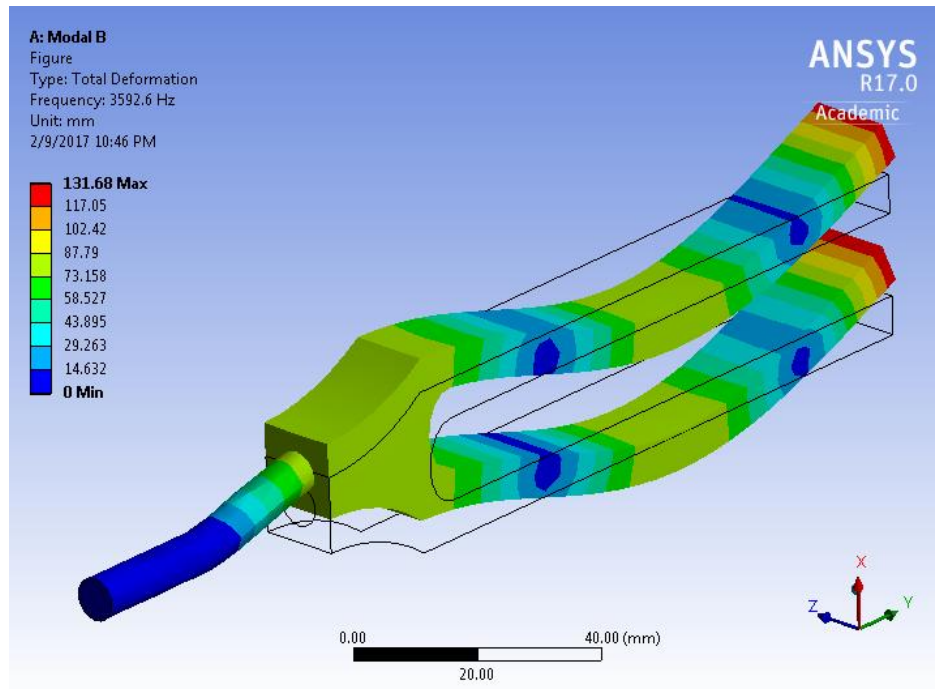


Figure A.11: Mode 9 for Case 1 (default mesh)

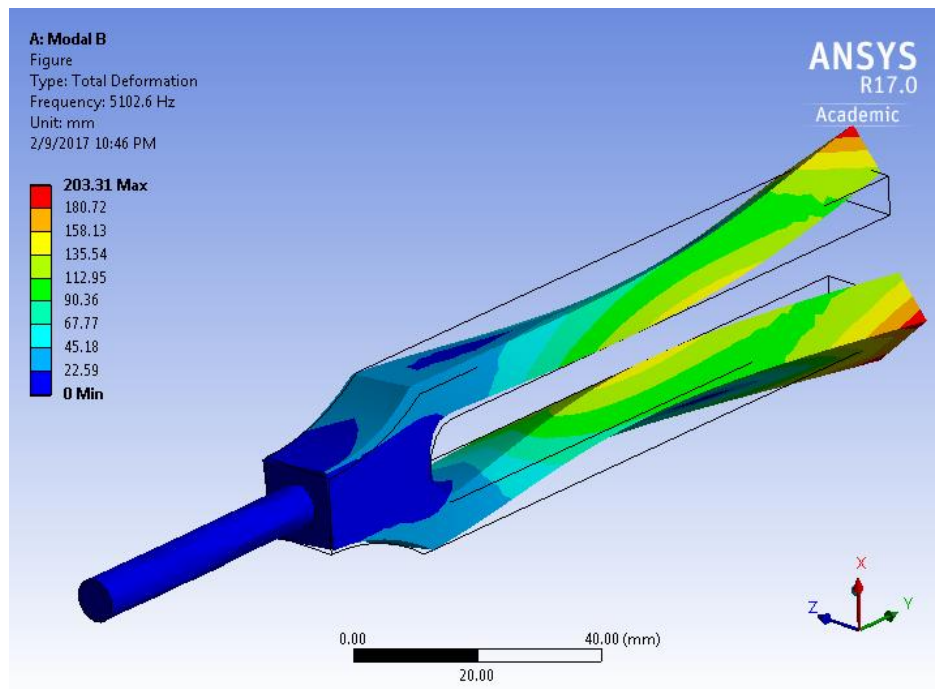


Figure A.12: Mode 10 for Case 1 (default mesh)

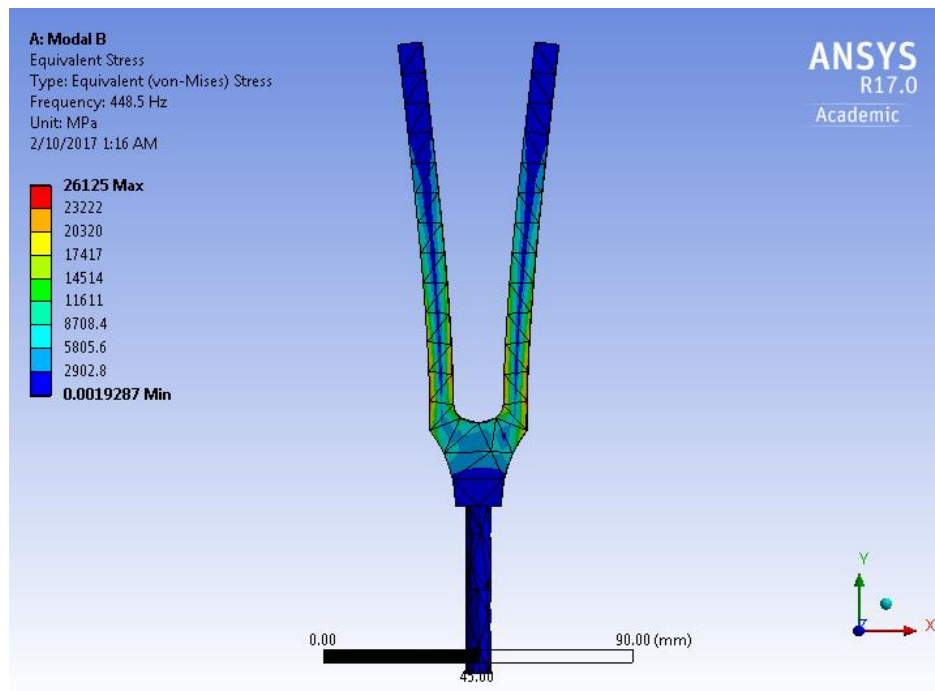


Figure A.13: Modal Stresses Mode 3 for Case 1 (default mesh)

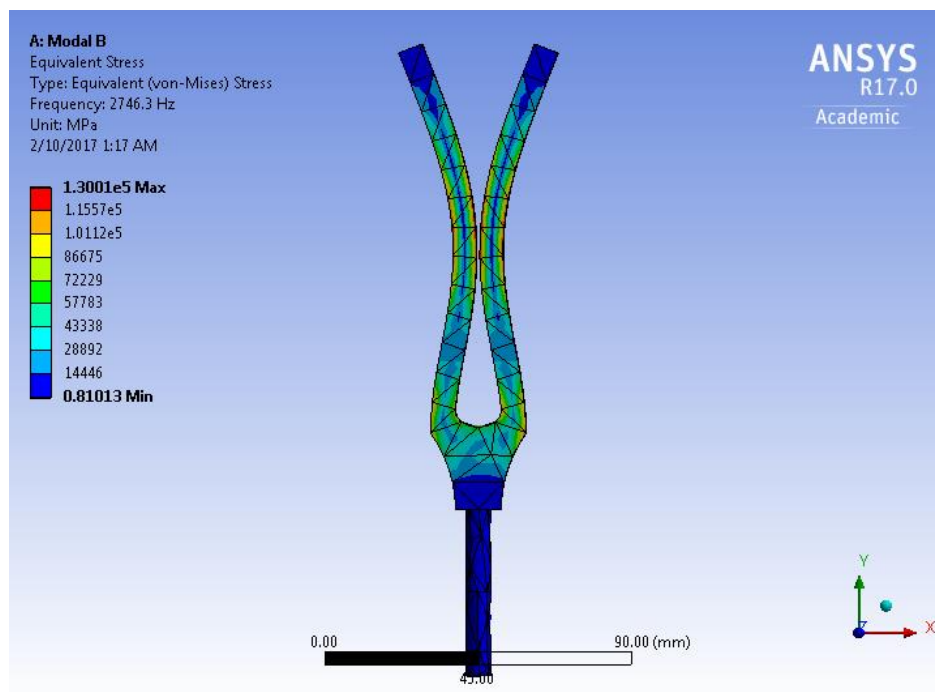


Figure A.14: Modal Stresses Mode 8 for Case 1 (default mesh)

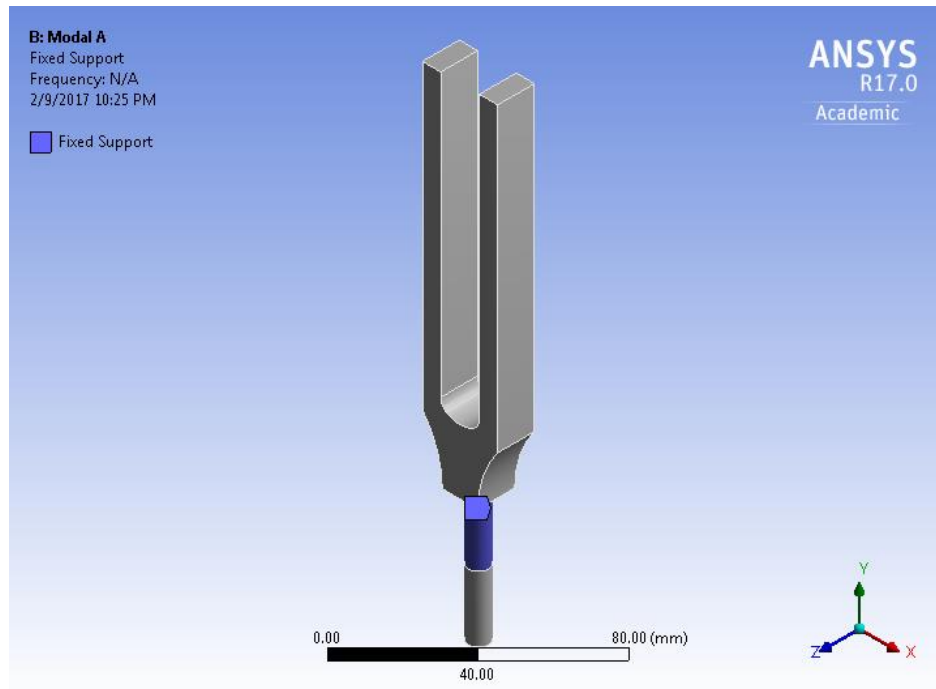


Figure A.15: BC, Fixed support, surface A (Case 2)

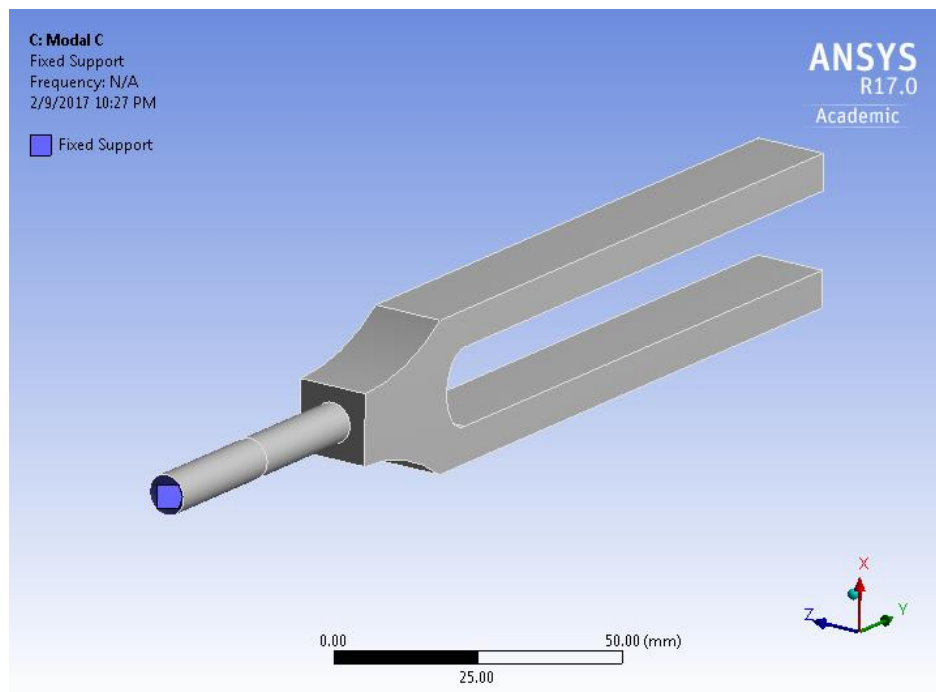


Figure A.16: BC, Fixed support, surface C (Case 3)

	Mode	✓ Frequency [Hz]
1	1.	372.09
2	2.	448.5
3	3.	500.64
4	4.	781.09
5	5.	1938.6
6	6.	2746.8
7	7.	3145.9
8	8.	4123.2
9	9.	4602.6
10	10.	5302.7

Figure A.17: Modes Frequency for Case 2 (default mesh)

	Mode	✓ Frequency [Hz]
1	1.	84.839
2	2.	86.169
3	3.	448.5
4	4.	565.24
5	5.	874.36
6	6.	1153.7
7	7.	1351.6
8	8.	2298.9
9	9.	2745.4
10	10.	3981.5

Figure A.18: Modes Frequency for Case 3 (default mesh)

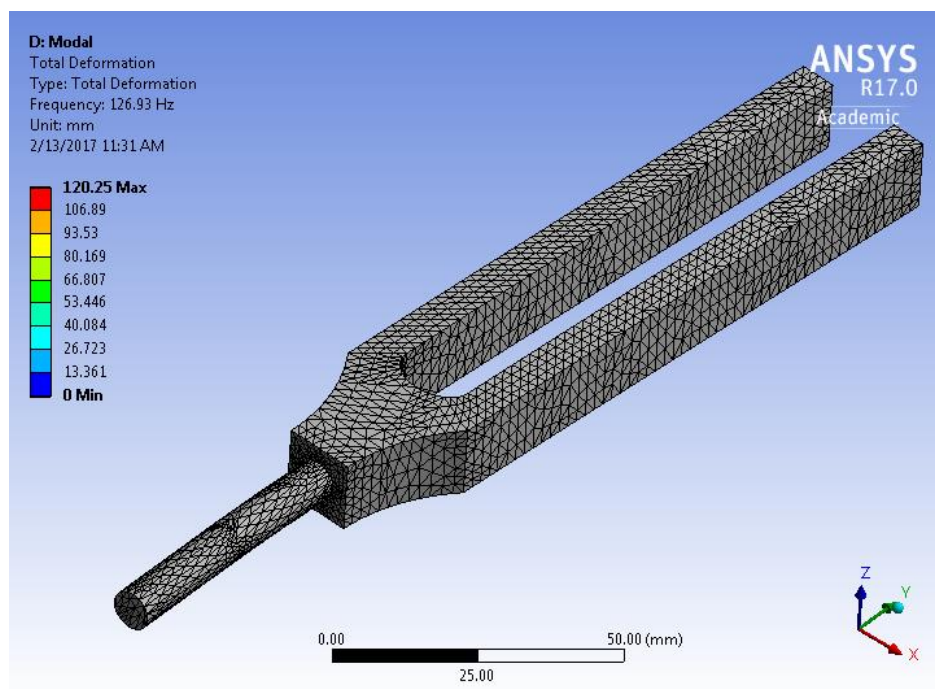
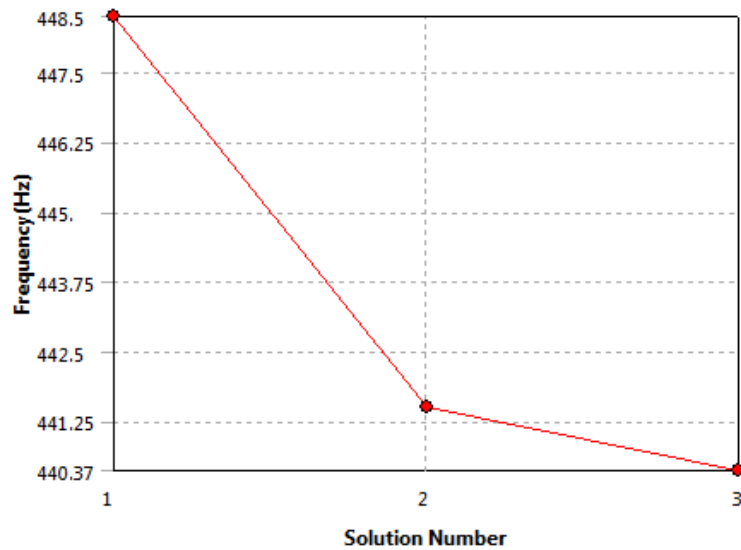


Figure A.19: Refined Mesh for Case 1

Convergence History



	Frequency(Hz)	Change (%)	Nodes	Elements
1	448.5		1044	455
2	441.5	-1.5721	6413	3606
3	440.37	-0.25709	44000	28497

Figure A.20: Convergence of Mode 3 for Case 1

	Mode	<input checked="" type="checkbox"/> Frequency [Hz]
1	1.	126.93
2	2.	130.79
3	3.	440.37
4	4.	643.33
5	5.	1126.9
6	6.	1548.6
7	7.	1850.9
8	8.	2699.3
9	9.	3398.6
10	10.	4787.5

Figure A.21: Modes Frequency for Case 1 (refined mesh)

	Mode	✓ Frequency [Hz]
1	1.	132.33
2	2.	137.89
3	3.	450.46
4	4.	659.33
5	5.	1155.1
6	6.	1592.7
7	7.	1960.
8	8.	2758.
9	9.	3611.6
10	10.	5089.5

Figure A.22: Modes Frequency for Case 1 (default mesh) using Aluminum

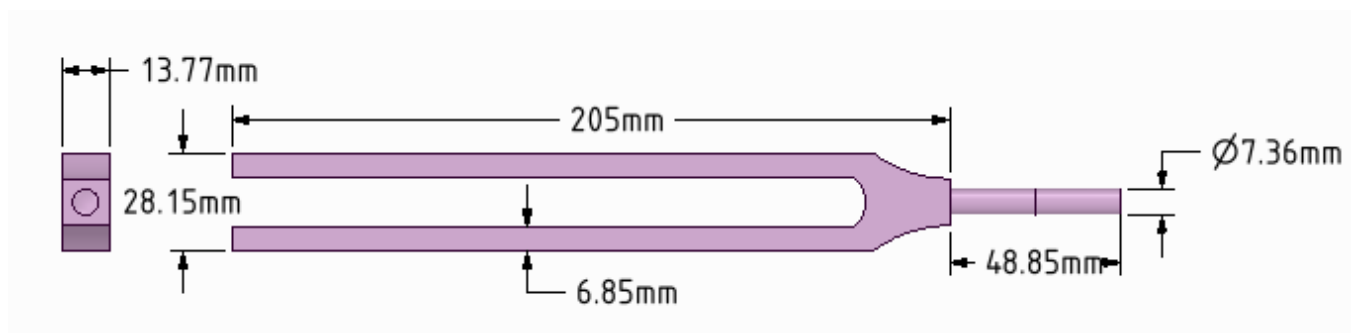


Figure A.23: Tuning fork dimensions (Design 2, Material: Copper Alloy)

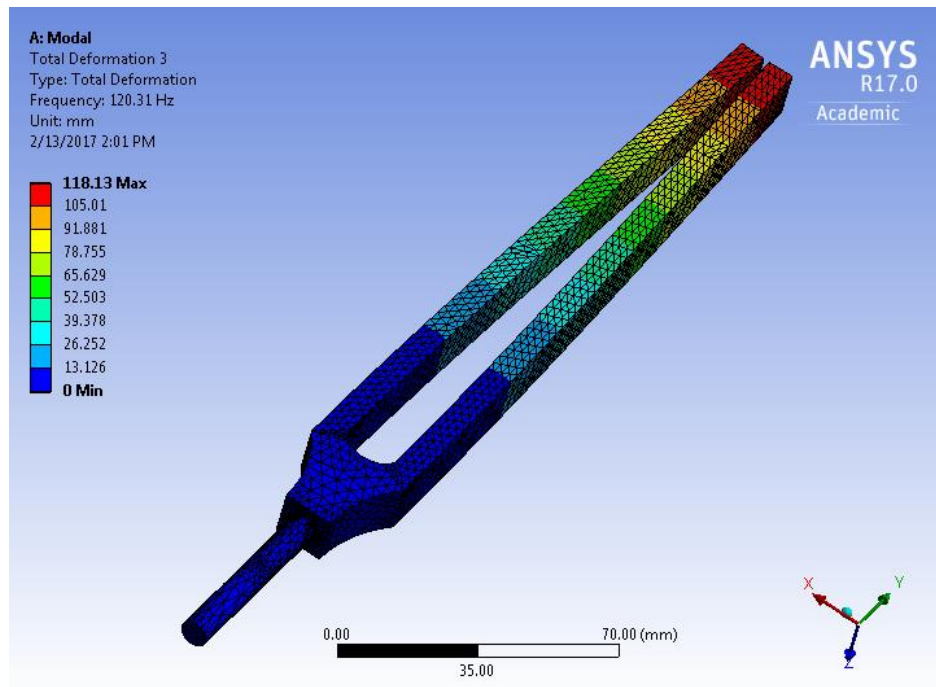
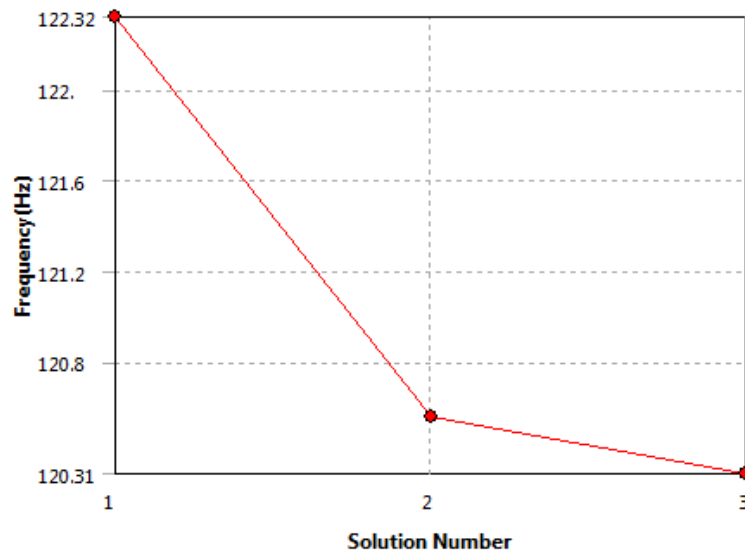


Figure A.24: Mode 3 of Vibration after mesh refinement (Design 2, Material: Copper Alloy)

	Mode	<input checked="" type="checkbox"/> Frequency [Hz]
1	1.	49.108
2	2.	52.062
3	3.	120.31
4	4.	212.19
5	5.	390.32
6	6.	711.64
7	7.	737.38
8	8.	747.8
9	9.	1173.
10	10.	1630.7

Figure A.25: Modes Frequency for Design 2, Material: Copper Alloy

Convergence History



	Frequency(Hz)	Change (%)	Nodes	Elements
1	122.32		980	389
2	120.56	-1.4428	5027	2630
3	120.31	-0.21191	33699	20953

Figure A.26: Convergence of Mode 3 for Design 2, Material: Copper Alloy

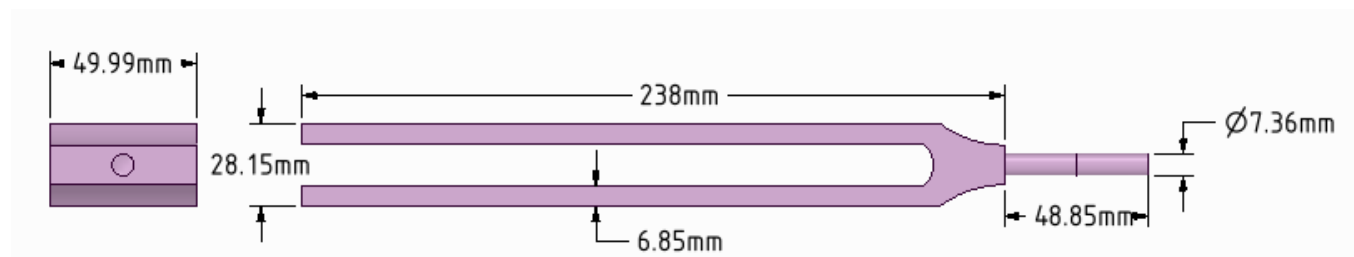


Figure A.27: Tuning fork dimensions (Design 3, Material: Aluminum Alloy)

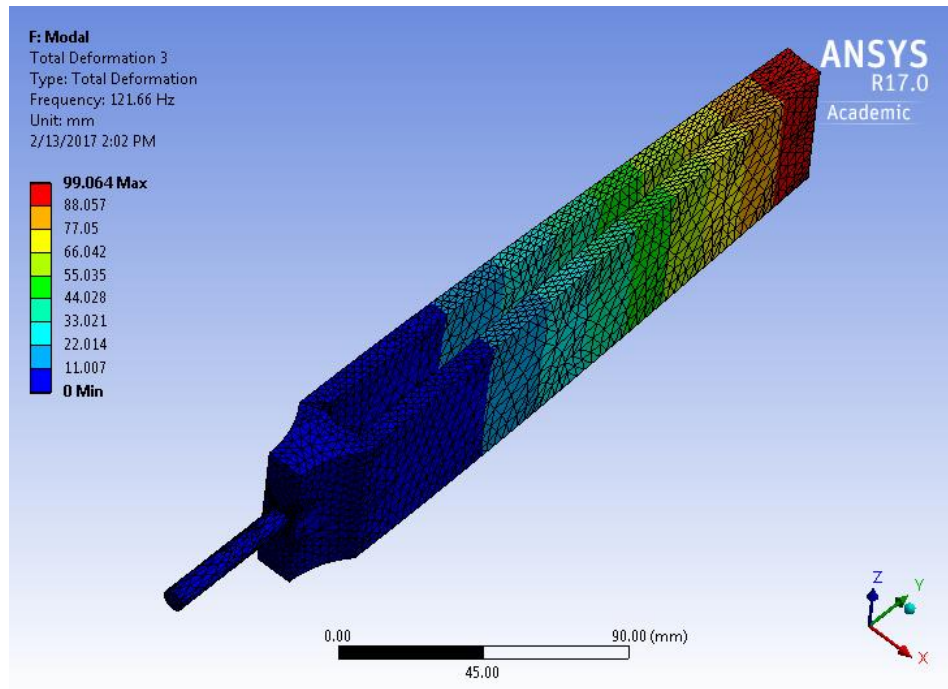
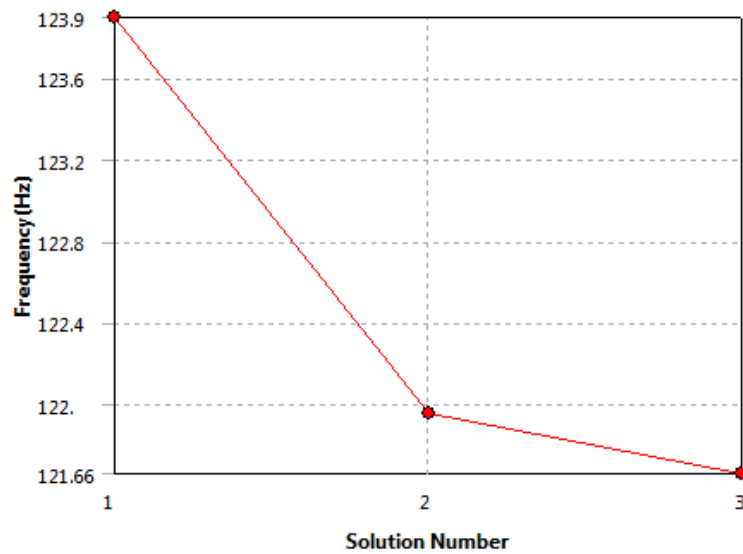


Figure A.28: Mode 3 of Vibration after mesh refinement (Design 3, Material: Aluminum Alloy)

	Mode	<input checked="" type="checkbox"/> Frequency [Hz]
1	1.	30.697
2	2.	31.541
3	3.	121.66
4	4.	221.98
5	5.	386.37
6	6.	683.29
7	7.	757.03
8	8.	990.7
9	9.	1083.9
10	10.	1157.9

Figure A.29: Modes Frequency for Design 3, Material: Aluminum Alloy

Convergence History



	Frequency(Hz)	Change (%)	Nodes	Elements
1	123.9		1371	578
2	121.96	-1.5793	7615	4154
3	121.66	-0.24496	47520	29978

Figure A.30: Convergence of Mode 3 for Design 3, Material: Aluminum Alloy