24-650 Applied Finite Element Analysis

Homework No 5
Fin design of a tube
Ignacio Cordova

The objective of this assignment was to perform a thermal-structural analysis of a steel finned tube subjected to freezing conditions and understand the thermal-structural performance of the tube as a function of the fin size.

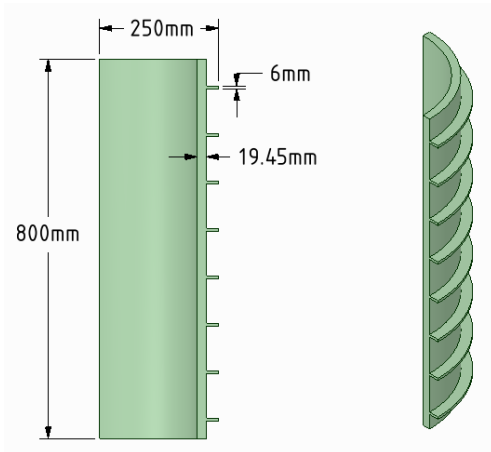

Figure 1: Finned tube dimensions

1. Setup

The first step was to import the geometry in Ansys Mechanical and create a Steady-State Thermal analysis. This is shown in Figure A.1 for the 3D analysis and Figure A.6 for the axisymmetric 2D analysis. For both models, I used a mesh with an element size of **10 mm**.

With the mesh ready, I created the following initial and boundary conditions (shown in Figure A.2 for the 3D model and Figure A.7 for the 2D model):

- Initial temperature: $0\,℃$
- Variable temperature in the Y direction for the inner surface of the pipe (from $100\,℃$ to $90\,℃$)
- Convection between the outer surface of the pipe and the air (stagnant air, simplified case at $0\,℃$)

After that, a Static Structural Analysis was done using the temperature results obtained in the Steady-State Thermal and considering an initial temperature of $0\,℃$. The boundary conditions are presented below and shown in:

- The Y direction is constrained at the bottom on the pipe.

2. Results and Analysis

The results obtained for the 3D model correspond to a quarter of the total pipe, so the heat values were multiplied by 4. In the table below, the results for the thermal simulation are shown.

Fin length (mm)	Volume (mm^3)	Total Heat (W)	Pipe Heat (W)	Fin Heat (W)	Min/Max Fin Temp (C)	Efficiency (W/mm^3)
25 (3D base)	2.28E+07	-819.32	-503.72	-315.592	89.254 / 99.992	3.58766E-05
25 (2D base)	2.28E+07	-819.33	-503.74	-315.6	89.255/98.995	3.58773E-05
315 (Min size 3000 W)	5.74E+07	-3006.3	-500.99	-2505.3	25.729/96.31	5.23883E-05
170 (Most efficient)	3.69E+07	-2362.7	-501.8	-1860.9	57.455/97.105	6.39587E-05

Table 1: Thermal results (Mesh element size:10 mm)

For the 3D model (Fin length: 25 mm) and 2D model (Fin length: 25 mm), the values shown in Table 1 can be validated by looking the results from Figure A.4 to Figure A.10.

For the Static Structural Analysis, there are many ways to constrain the tube:

- Use a Symmetry Region element and use a Displacement Support constrained in Y for the base of the tube.
- Use a Symmetry Region element and use a Frictionless Support for the base of the tube.
- Use a Symmetry Region element and use a Remote Displacement Support constrained in Y for the base of the tube.

The one selected is shown in Figure A.11.

The selection of the 10 mm for the mesh element size was obtained after an optimization of the mesh. This was done by using the mesh element size as a parameter and giving different values until the variation for the maximum Equivalent Stress was near 1%. This is shown below:

Mesh Element Size (mm)	Max Equivalent Stress (Mpa)	%Difference
Default	3.43	-
30.00	3.17	-7.73
20.00	3.16	-0.43
15.00	3.20	1.53
10.00	3.17	-1.08
5.00	3.20	0.89

Table 2: Correct Mesh selection

The Structural results are shown in Table 3. The first two rows are the results using a mesh element size of 10 mm for the 3D and 2D model. In both cases, the only external load is the thermal result presented in Table 1.

The critical location for both models is at the tip of the fin:

- Shown in Figure A.13 for Thermal 3D (Fin length: 25 mm)
- Shown in Figure A.19 for Thermal 2D (Fin length: 25 mm)

The state of stress for the critical location is circumferential, as it can be seen in the results shown in Table 3. To check for singularities, a refinement of the mesh was created for the sharp edge at the bottom of one of the fins. This is shown in Figure A.44. As can be seen, no singularities were found for the 3D model.

Fin length (mm)	**Load Case**	**Max USUM Displacement (mm)**	**Max Thermal Strain**	**Radial Stress (Mpa)**	**Hoop Strees (Mpa)**	**Equivalent Stress (Mpa)**
25	Thermal 3D	0.94305	1.20E-03	-0.20121	3.1676	3.1689
25	Thermal	0.94305	1.20E-03	-0.21764	3.1643	3.1651
315	Thermal	0.98226	1.20E-03	-54.209	61.321	69.593
315	G loading	1.6094	-	-53.931	18.365	51.103
315	Combined	1.6152	1.20E-03	-108.14	-73.014	101.02

Table 3: Structural results

For the selection of the minimum fin length for a Total Heat transfer of 3,000 W and for the selection of the most efficient fin length, I created a parameter named Fin Length, which went from 25 mm to 375 mm (mesh element size of 10 mm). For each value (12 in total), I calculated the Total Heat and the Volume. In the Figure 2 is shown that for a Total Heat of 3,000 W, we need at least a Fin length of **315 mm** (Total Heat=3,006.28 W). For the most efficient point, we can see in the Figure 3 that the peak is when the Fin length is around **170 mm** (Efficiency: 6.3958 E-05). The difference between lengths measured is about 50 mm, but for lengths close to 315 mm and 170 mm, I added more points to be sure that those points selected where correct.

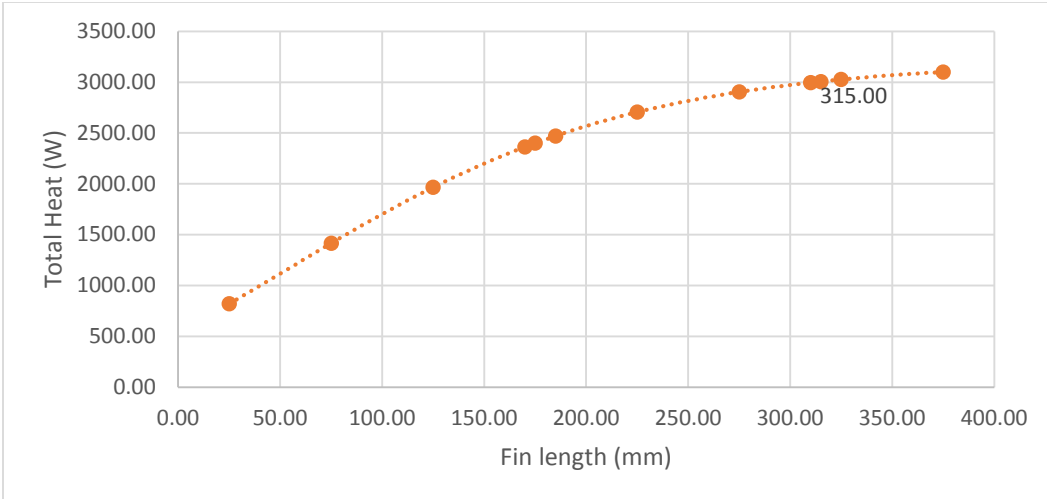

Figure 2: Total Heat Vs Fin length (10 mm element size mesh)

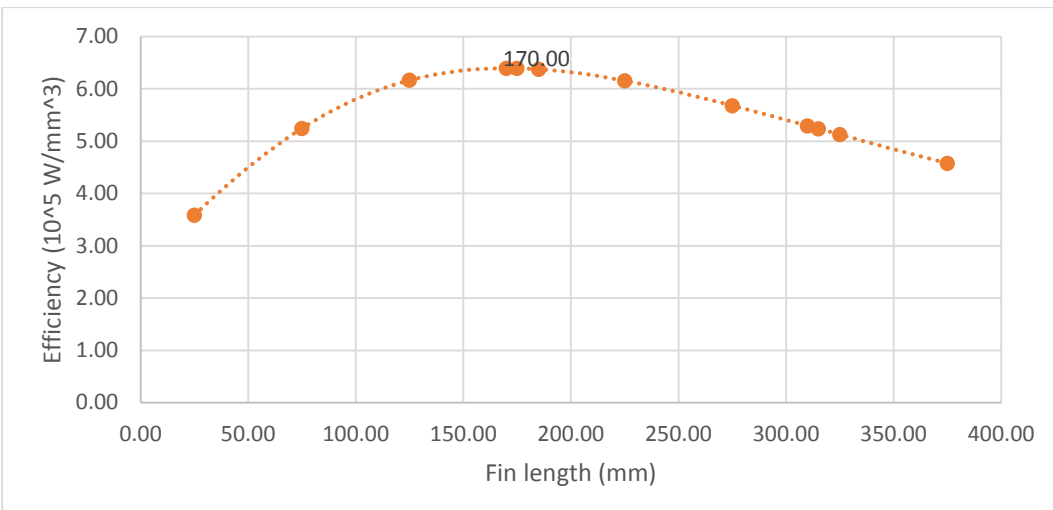

Figure 3: Efficiency Vs Fin length (10 mm element size mesh)

To understand the physics behind this problem, we need to understand the equations involved. If we assume a one-dimensional, steady-state conduction for a fin with conductivity (k), uniform cross-sectional area (A_c), perimeter (P) and negligible generation and radiation, we have that:

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0$$

If we define $m^2 = \frac{hP}{kA_c}$ and $\theta = (T - T_\infty)$, we have a new equation:

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

The solution for that equation is shown in Figure 4. It can be seen that the temperature difference decrease in a non-linear way, so the heat transferred to the surroundings by convection it is bigger at the beginning of the fin and starts decreasing until a certain distance x, where the heat transferred is near zero. If the fin length is too large, it means that there is bigger portion that is transferring almost zero heat, so the efficiency drops.

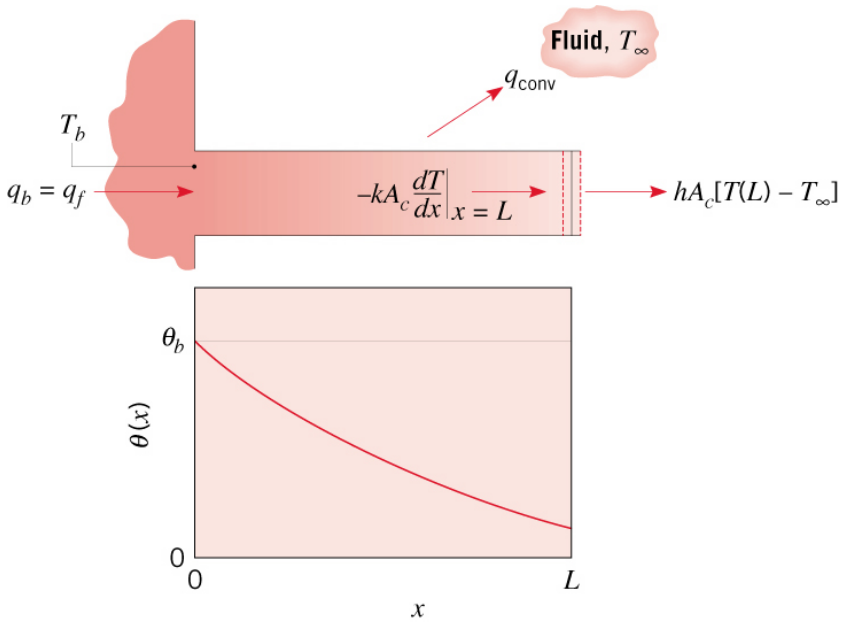

Figure 4: Heat Transfer in an extended surface[1]

As it was shown in Figure 2, the minimum fin length required to transfer a total of 3,000 W is 315 mm. Using that value and using a mesh element size of 5 mm, a new Static Structural analysis was done. The results are shown in the third row of the Table 3 and from Figure A.27 to Figure A.32. It can be seen that for the fin length of 315 mm, the Equivalent Stress is 23 times bigger than the Equivalent Stress for a fin length of 25 mm. It also can be seen that the critical location is not at the tip of the fin but around 170 mm from the bottom (Figure 5).

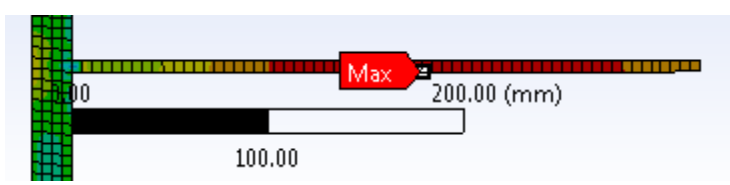

Figure 5: Equivalent Stress critical location for Fin length=315 mm

The location of the critical point coincides with the location of the most efficient length for the fin. The explanation of this is that the most efficient length is the one that transfers more heat, so the stress is max in that location.

[1] Obtained from http://www.me.nchu.edu.tw/lab/lab516/2014/Heat%20Transfer-PDF-Incropera-1/3c.pdf

The state of stress for the critical location is circumferential, as it can be seen in the results shown in Table 3.

Using the same mesh element size of 5 mm, a new Static Structural analysis was done but without the thermal load and now considering a 10G acceleration load in the Y direction. The results are shown in the fourth row of the Table 3 and from Figure A.33 to Figure A.37. It also can be seen that the critical location is near to the bottom of the fin (Figure A.35). The state of stress for the critical location is radial, as can be seen in the results shown in Table 3.

Finally, a Solution Combination analysis with the thermal load and the 10G acceleration load was done for the same geometry (mesh element size of 5 mm). The results are shown in the fifth row of the Table 3 and from Figure A.38 to Figure A.43. It also can be seen that the critical location is near to the bottom of the fin (Figure A.40). The state of stress for the critical location is mostly radial, as can be seen in the results shown in Table 3.

To check for singularities, three refinements of the mesh were done for the sharp corners at the bottom of one of the fins. This is shown in Figure 6. As can be seen, for the Thermal Load case, there is a big increase of the equivalent stress at the sharp corners when the refinement increases. That means that a pure Thermal Load can produce a singularity at the sharp corners positions. This is also shown in Figure A.45

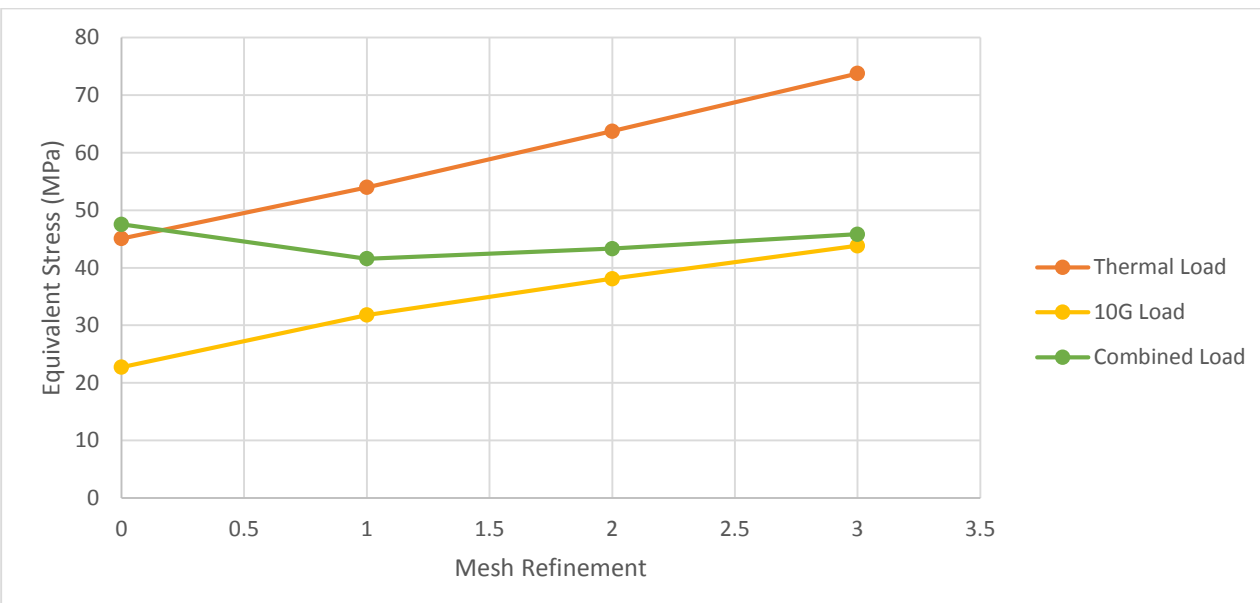

Figure 6: Mesh Refinement to find singularities

3. Appendix

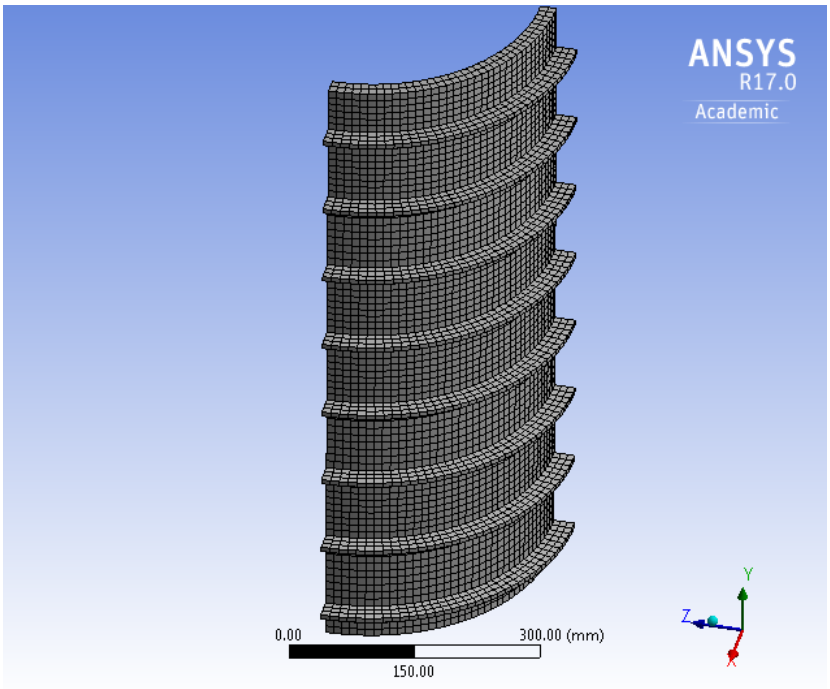

Figure A.1: 3D model mesh, Element Size 10 mm (Fin length: 25 mm)

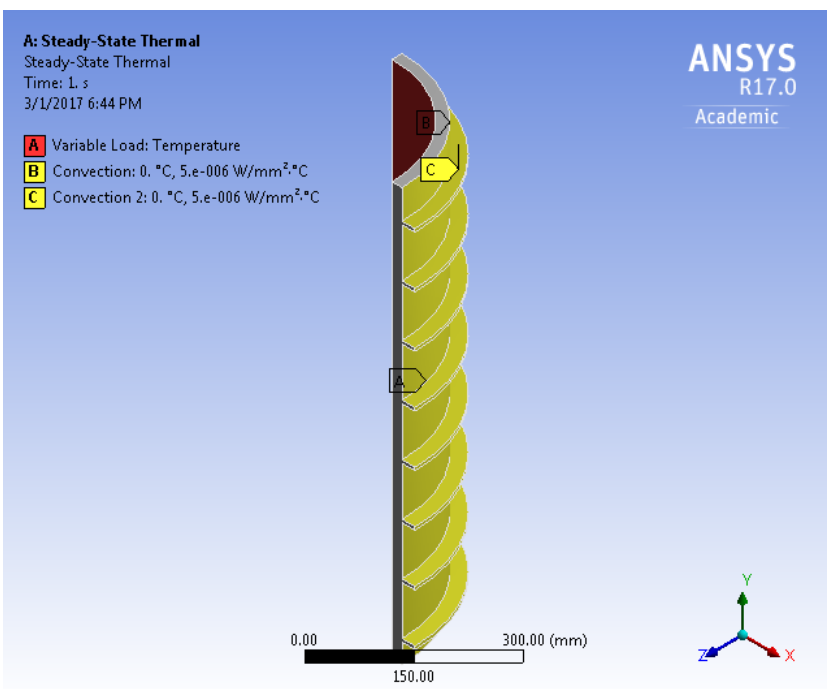

Figure A.2: Boundary Conditions, Steady-State Thermal 3D (Fin length: 25 mm)

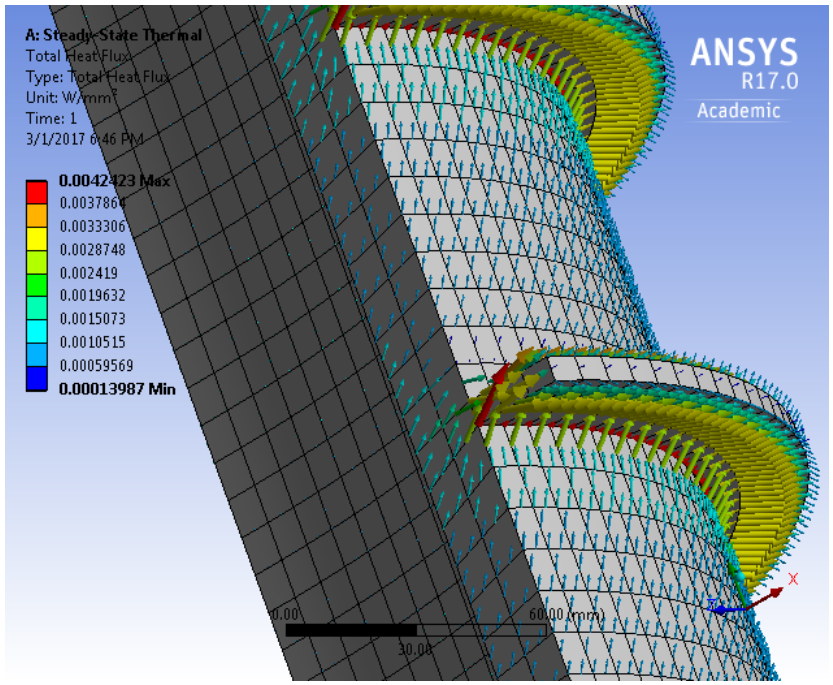

Figure A.3: Heat Flux, Steady-State Thermal 3D (Fin length: 25 mm)

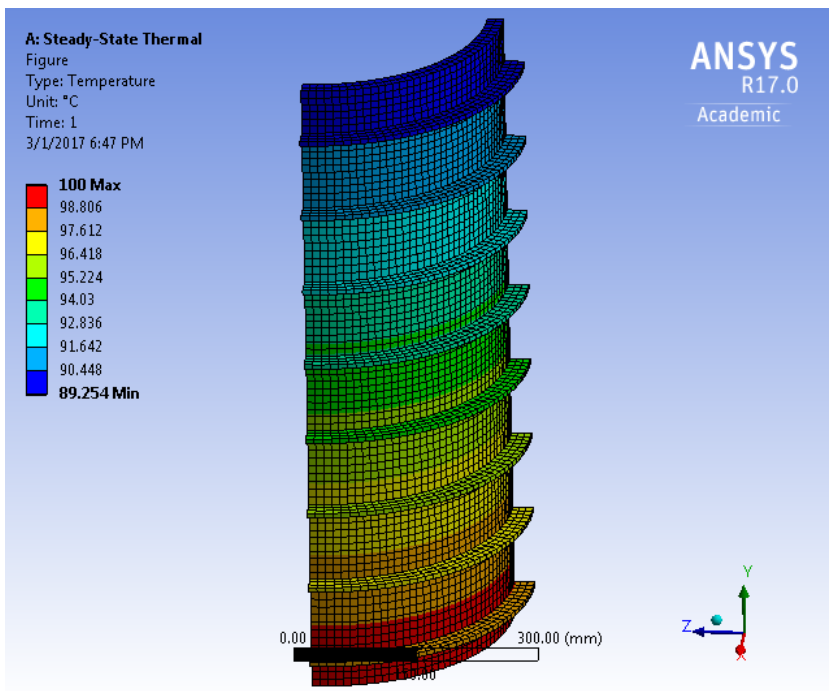

Figure A.4: Temperature, Steady-State Thermal 3D (Fin length: 25 mm)

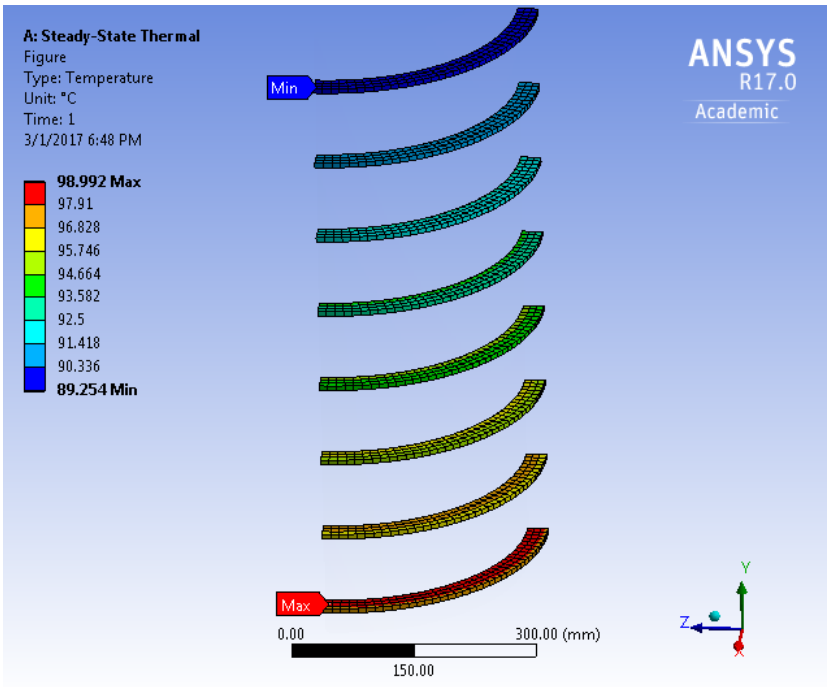

Figure A.5: Fins Temperature, Steady-State Thermal 3D (Fin length: 25 mm)

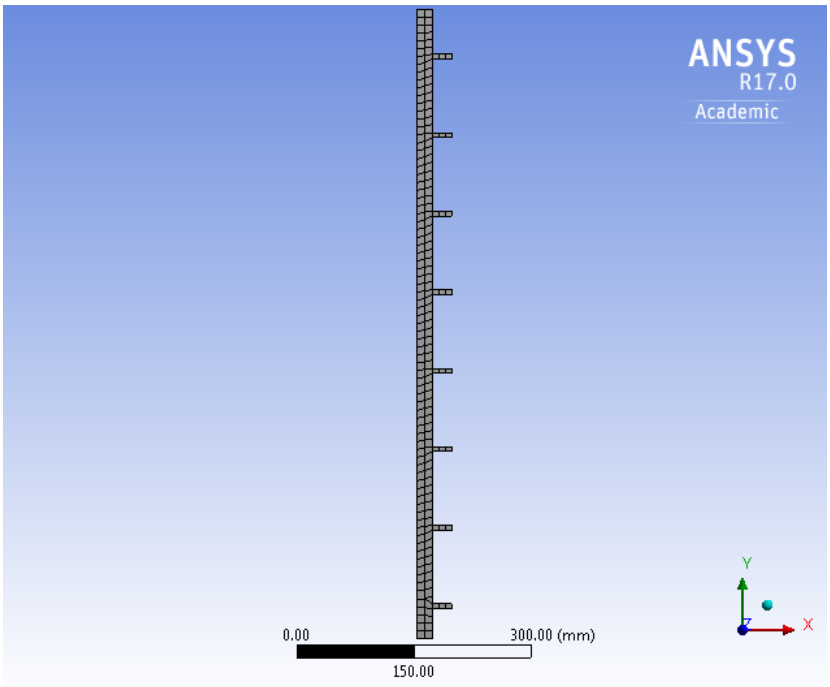

Figure A.6 2D model mesh, Element Size 10 mm (Fin length: 25 mm)

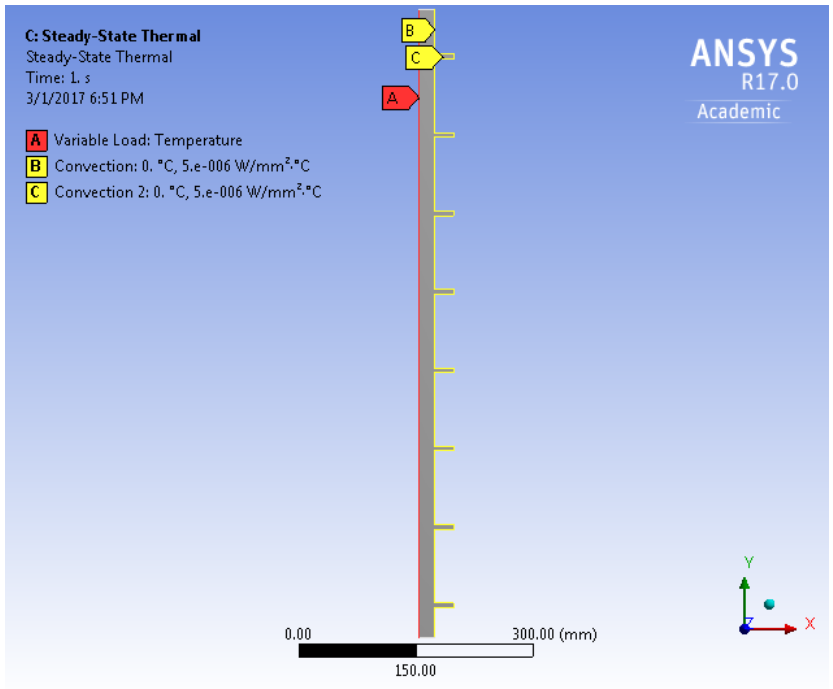

Figure A.7: Boundary Conditions, Steady-State Thermal 2D (Fin length: 25 mm)

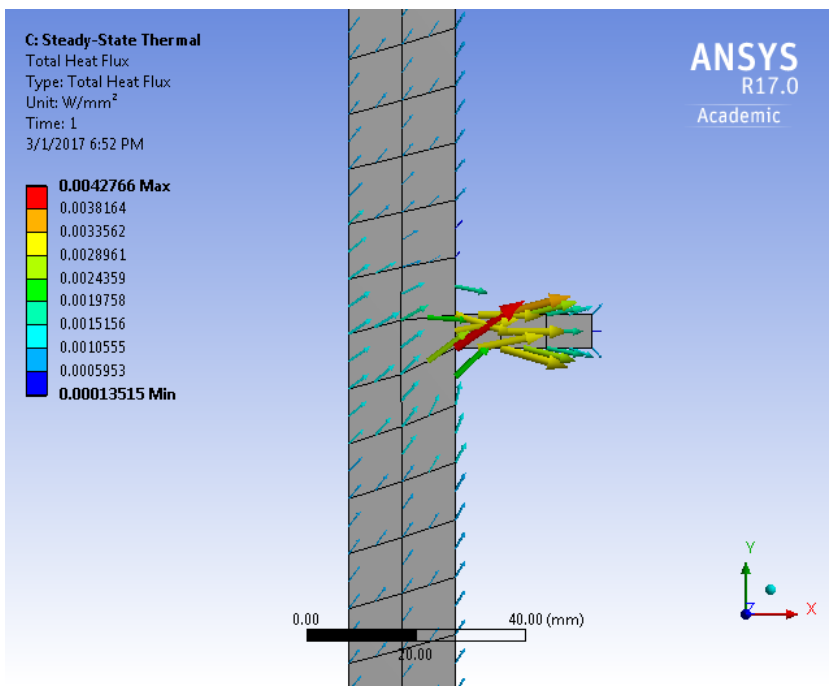

Figure A.8: Heat Flux, Steady-State Thermal 2D (Fin length: 25 mm)

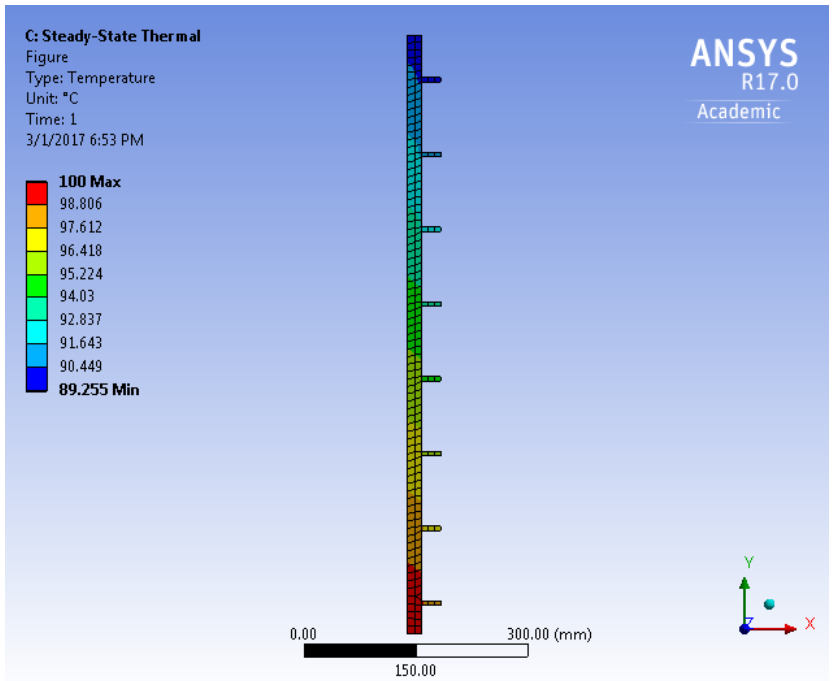

Figure A.9: Temperature, Steady-State Thermal 2D (Fin length: 25 mm)

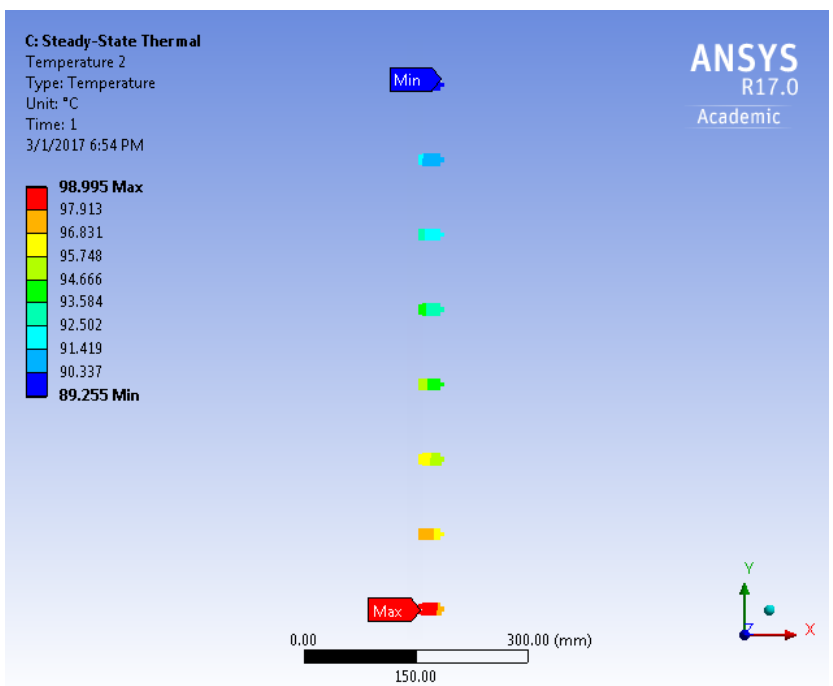

Figure A.10: Fins Temperature, Steady-State Thermal 2D (Fin length: 25 mm)

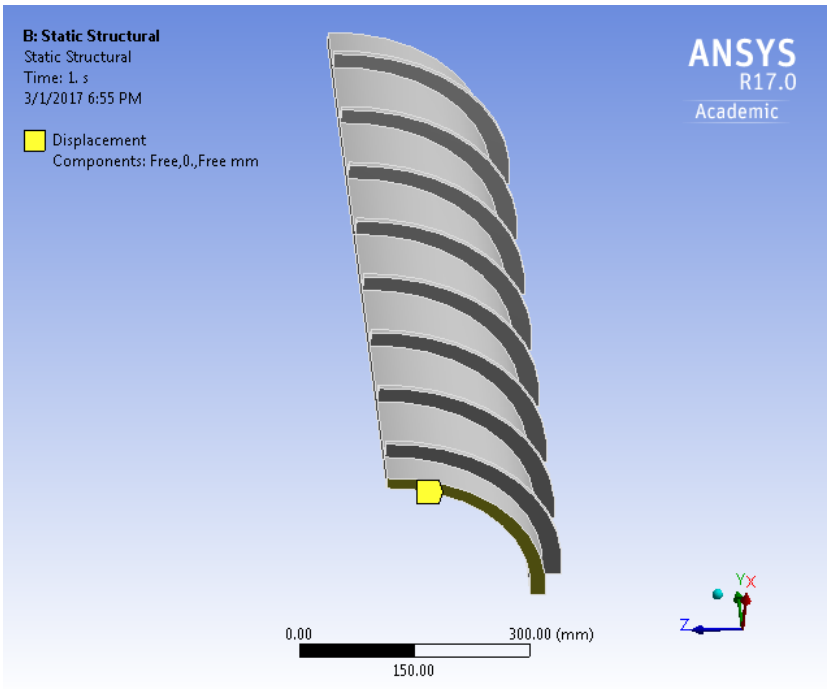

Figure A.11: Boundary Conditions, Static Structural with thermal load 3D (Fin length: 25 mm)

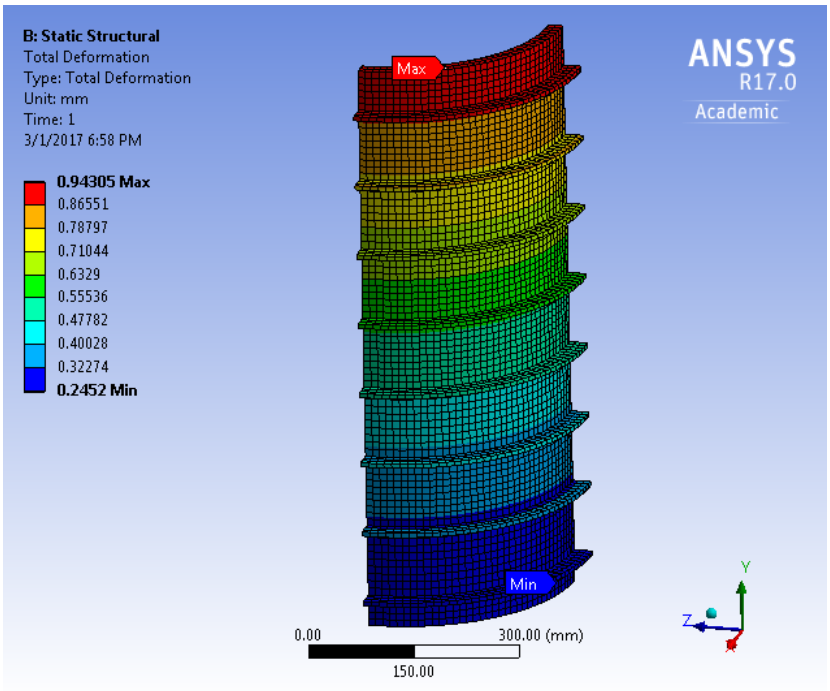

Figure A.12: Total Deformation, Static Structural with thermal load 3D (Fin length: 25 mm)

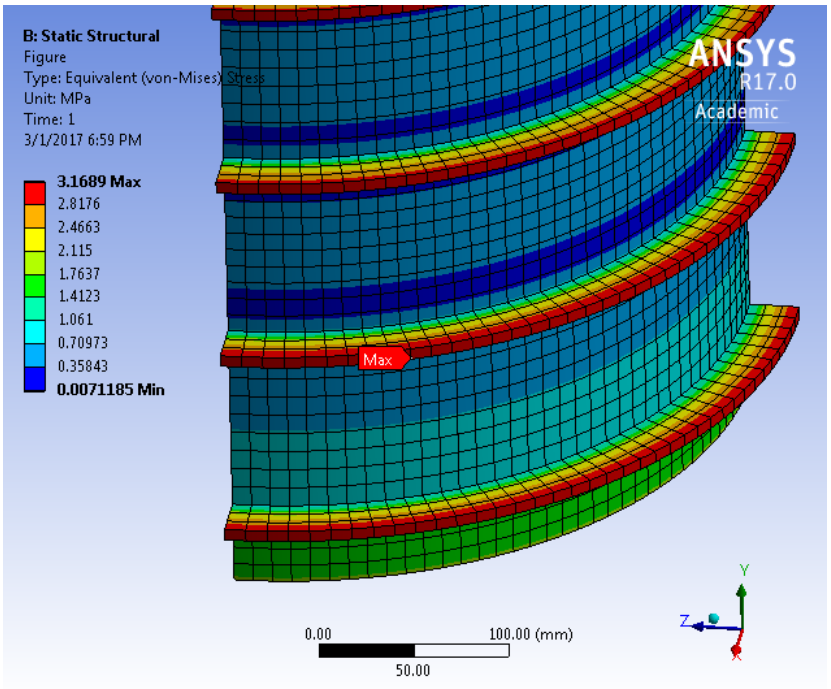

Figure A.13: Equivalent Stress, Static Structural with thermal load 3D (Fin length: 25 mm)

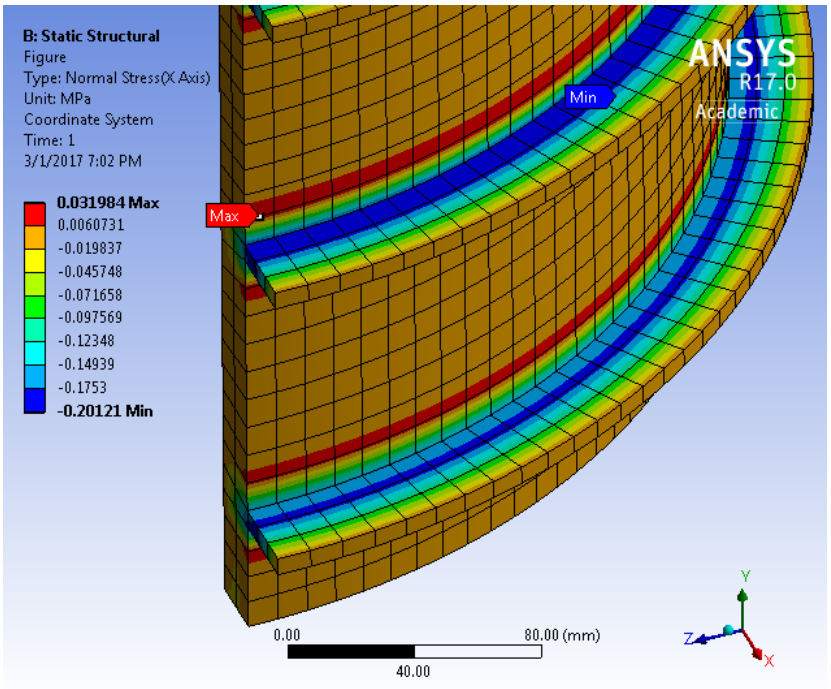

Figure A.14: Radial Stress, Static Structural with thermal load 3D (Fin length: 25 mm)

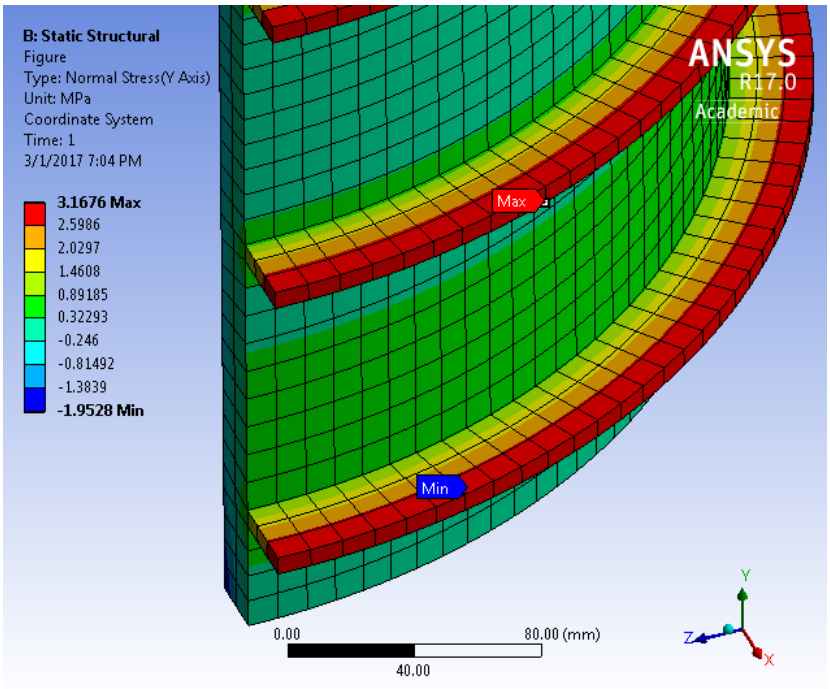

Figure A.15: Hoop Stress, Static Structural with thermal load 3D (Fin length: 25 mm)

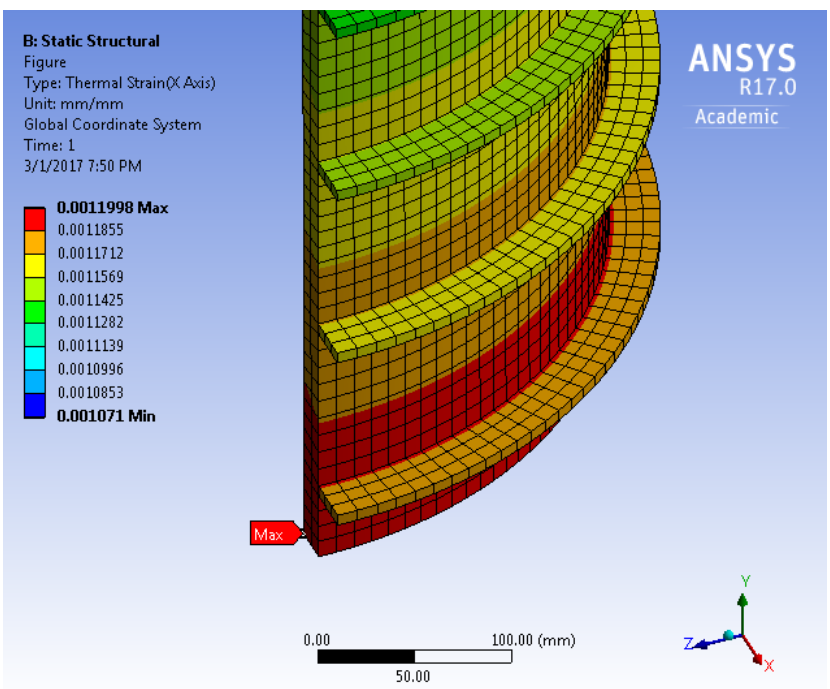

Figure A.16: Thermal Strain, Static Structural with thermal load 3D (Fin length: 25 mm)

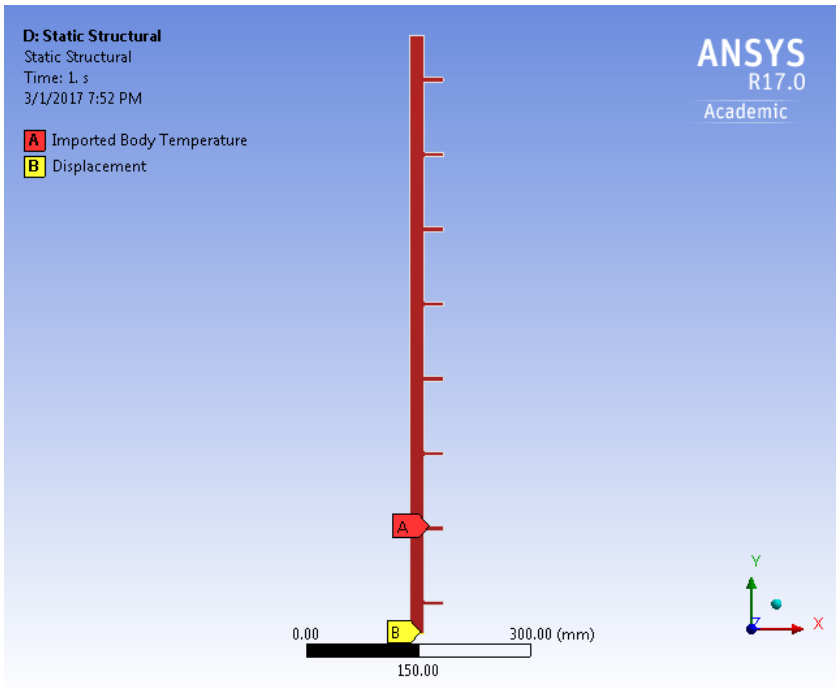

Figure A.17: Boundary Conditions, Static Structural with thermal load 2D (Fin length: 25 mm)

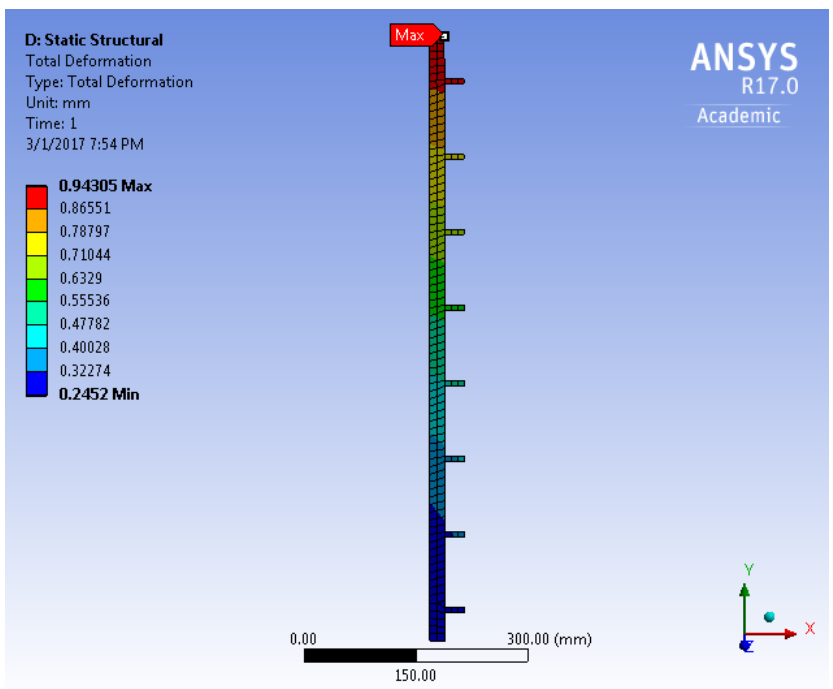

Figure A.18: Total Deformation, Static Structural with thermal load 2D (Fin length: 25 mm)

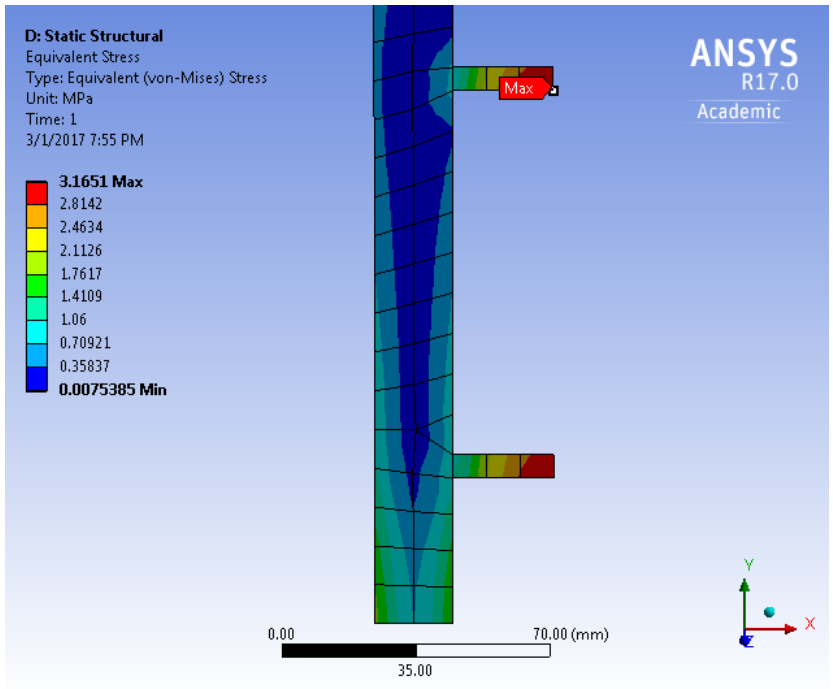

Figure A.19: Equivalent Stress, Static Structural with thermal load 2D (Fin length: 25 mm)

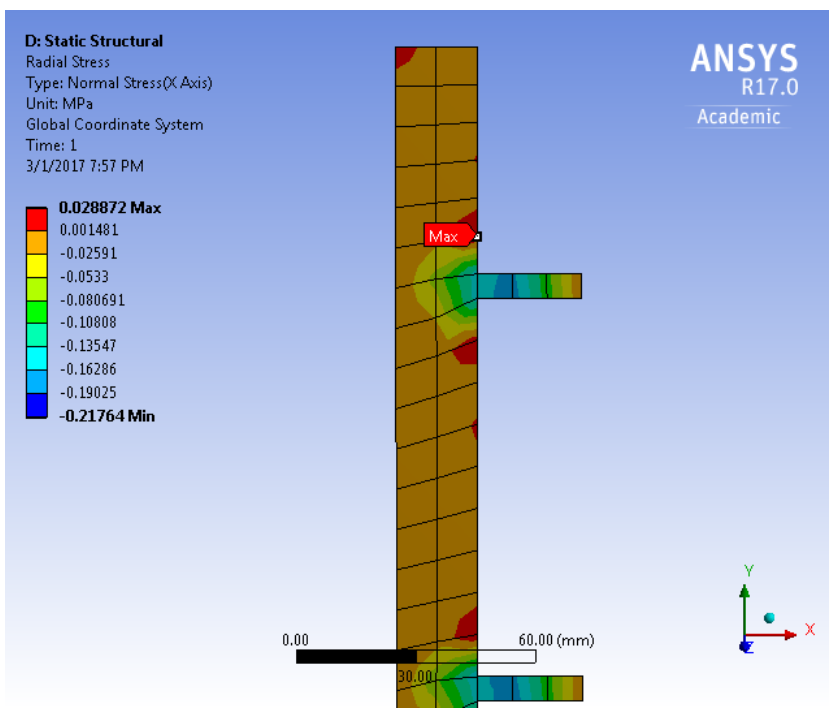

Figure A.20: Radial Stress, Static Structural with thermal load 2D (Fin length: 25 mm)

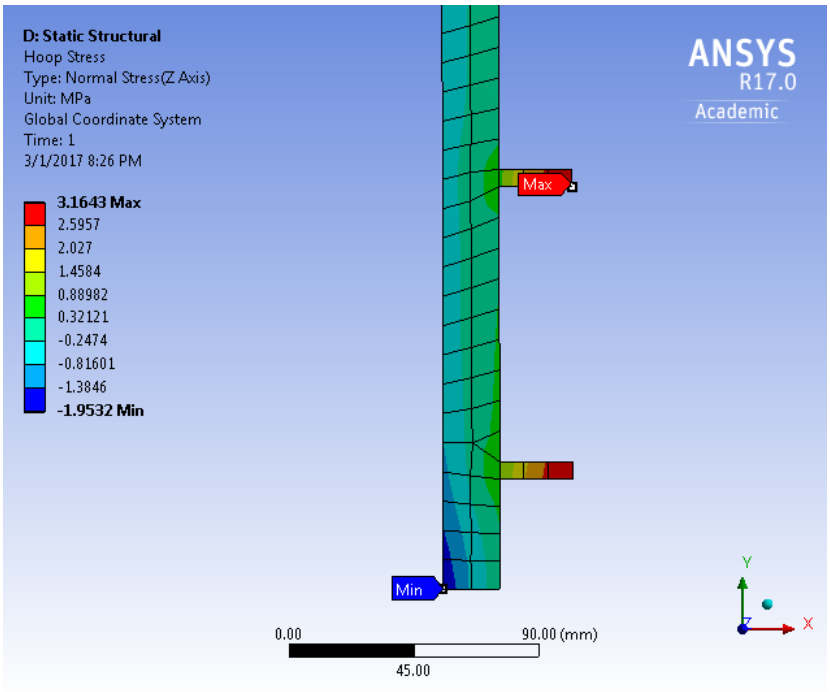

Figure A.21: Hoop Stress, Static Structural with thermal load 2D (Fin length: 25 mm)

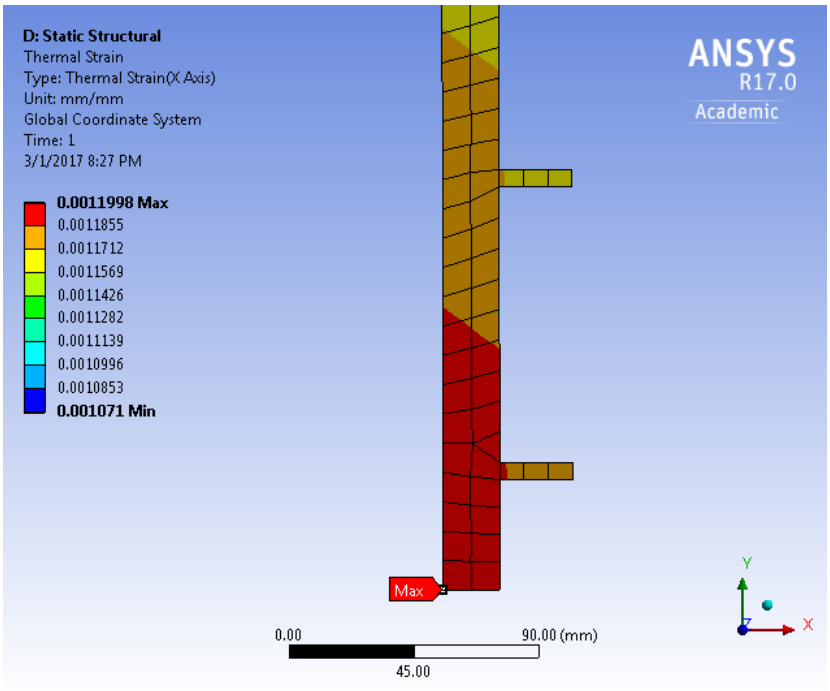

Figure A.22: Thermal Strain, Static Structural with thermal load 2D (Fin length: 25 mm)

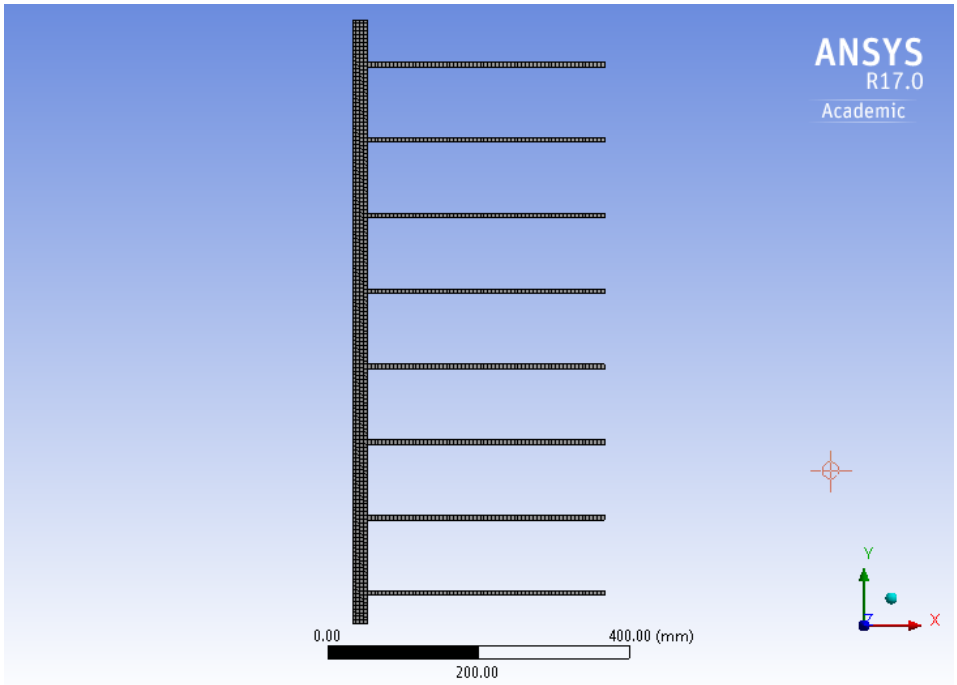

Figure A.23 2D model mesh, Element Size 5 mm (Fin length: 315 mm)

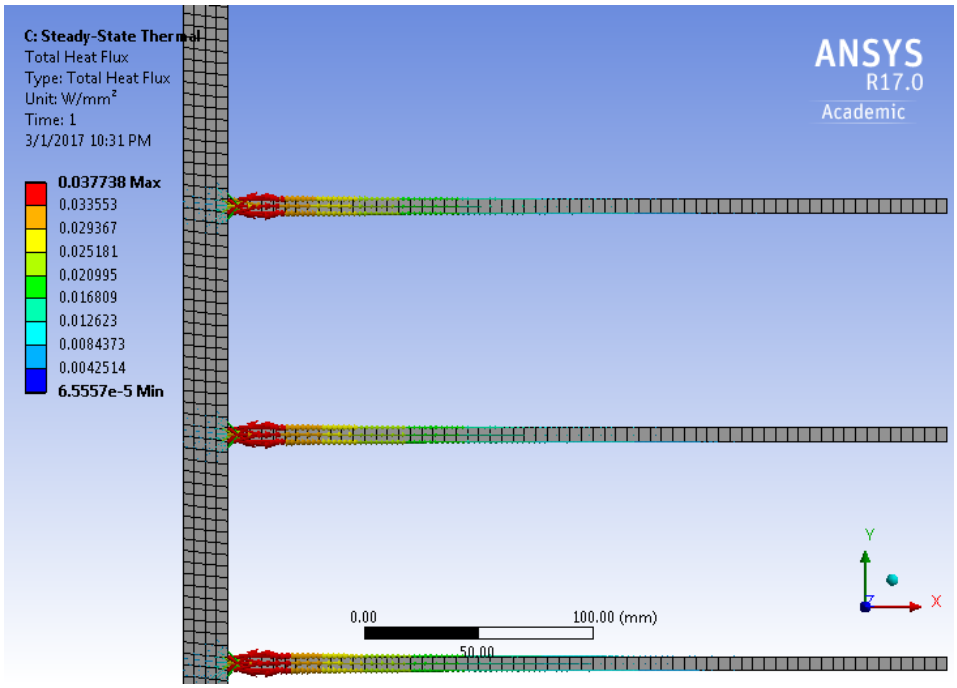

Figure A.24: Heat Flux, Steady-State Thermal 2D (Fin length: 315 mm)

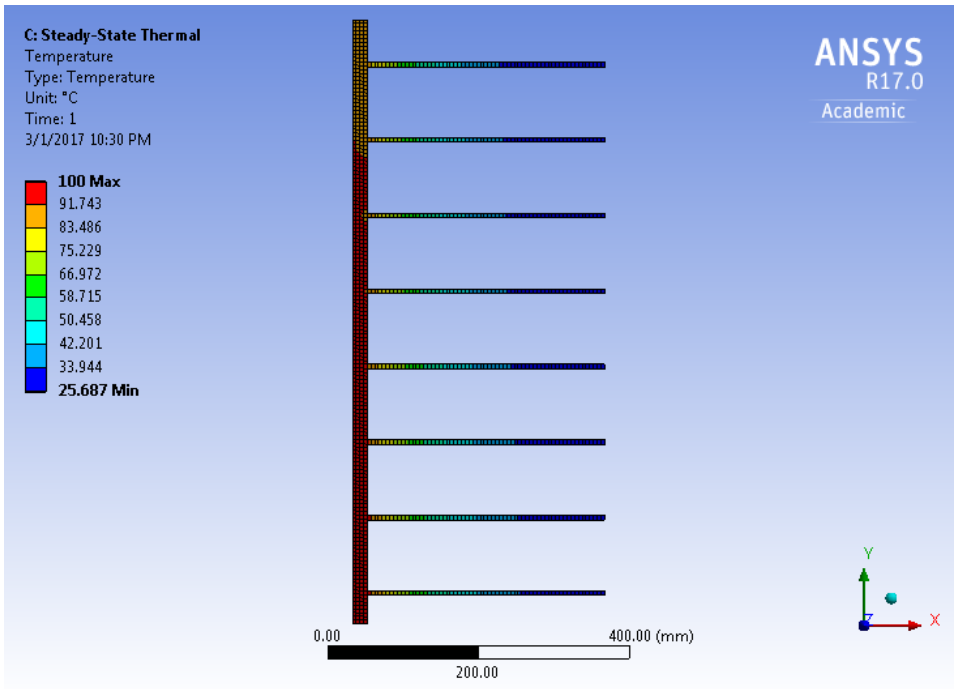

Figure A.25: Temperature, Steady-State Thermal 2D (Fin length: 315 mm)

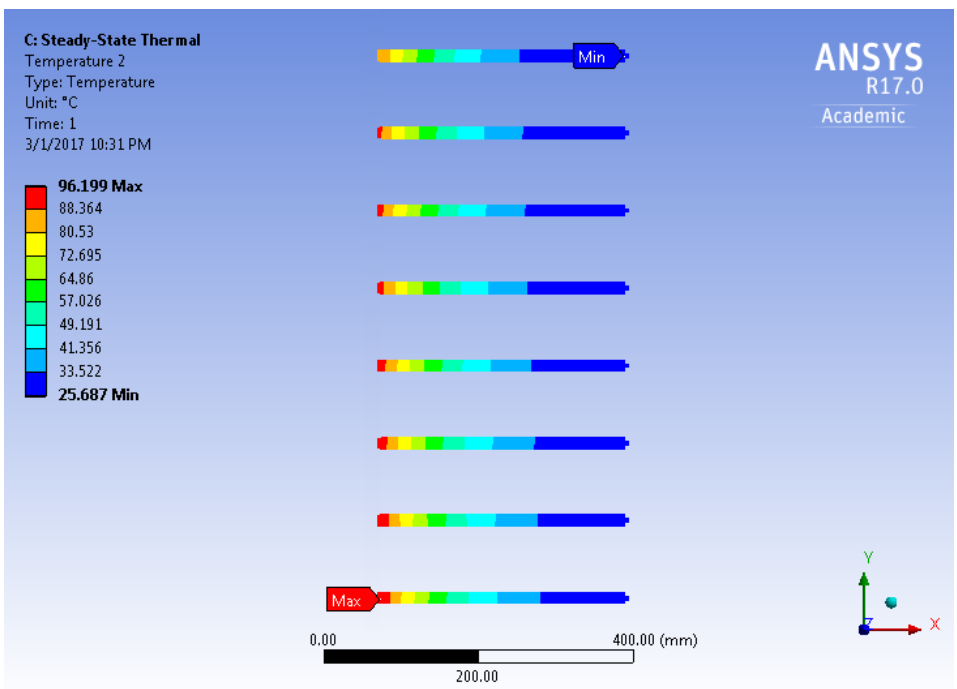

Figure A.26: Fins Temperature, Steady-State Thermal 2D (Fin length: 315 mm)

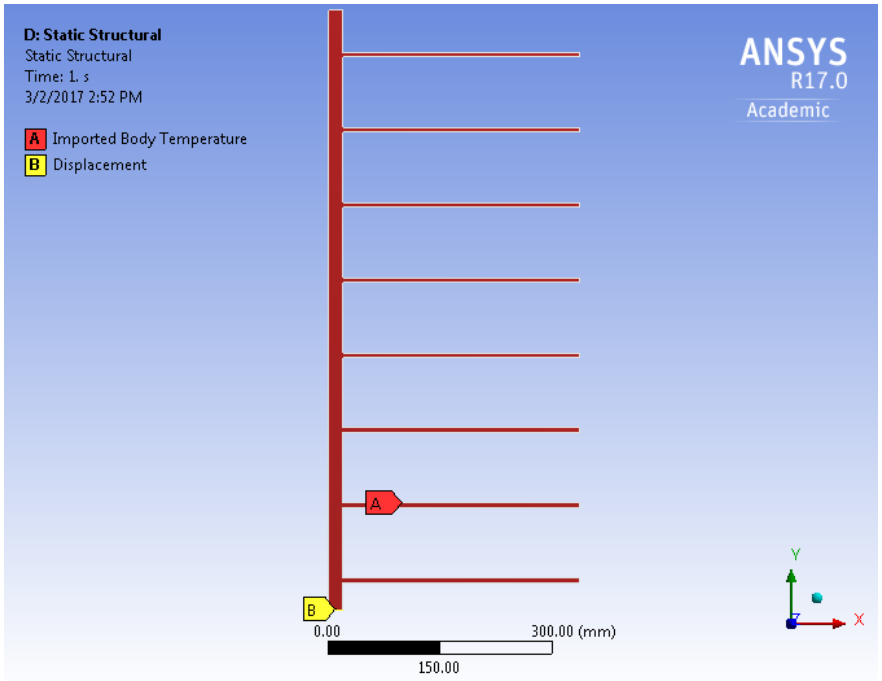

Figure A.27: Boundary Conditions, Static Structural with thermal load 2D (Fin length: 315 mm)

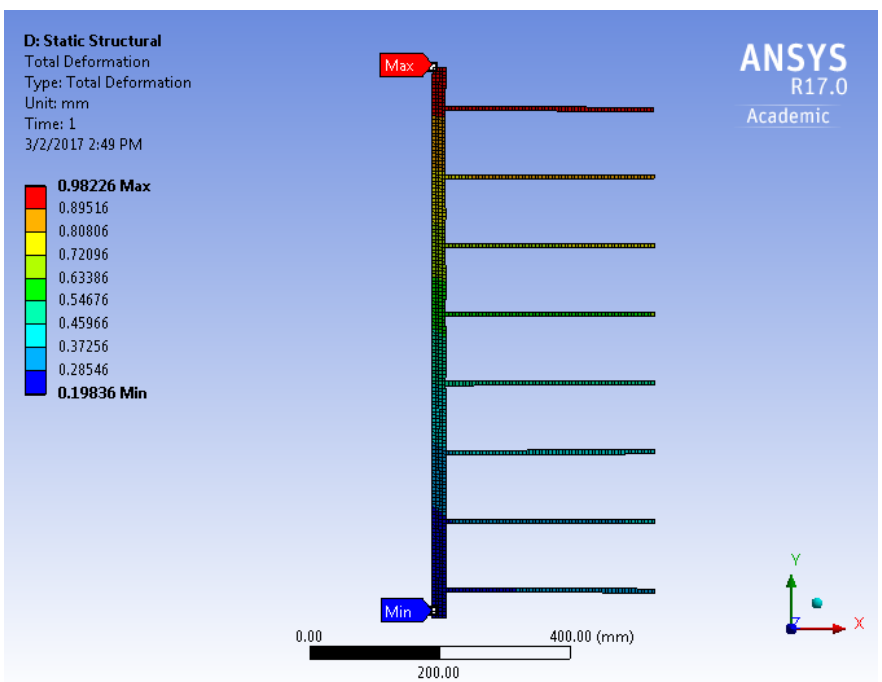

Figure A.28: Total Deformation, Static Structural with thermal load 2D (Fin length: 315 mm)

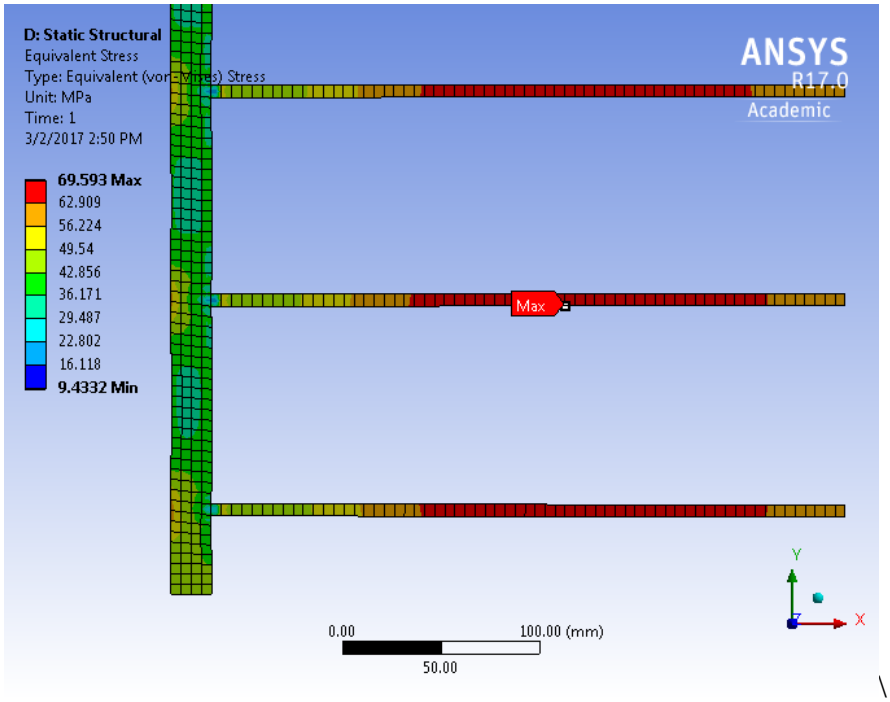

Figure A.29: Equivalent Stress, Static Structural with thermal load 2D (Fin length: 315 mm)

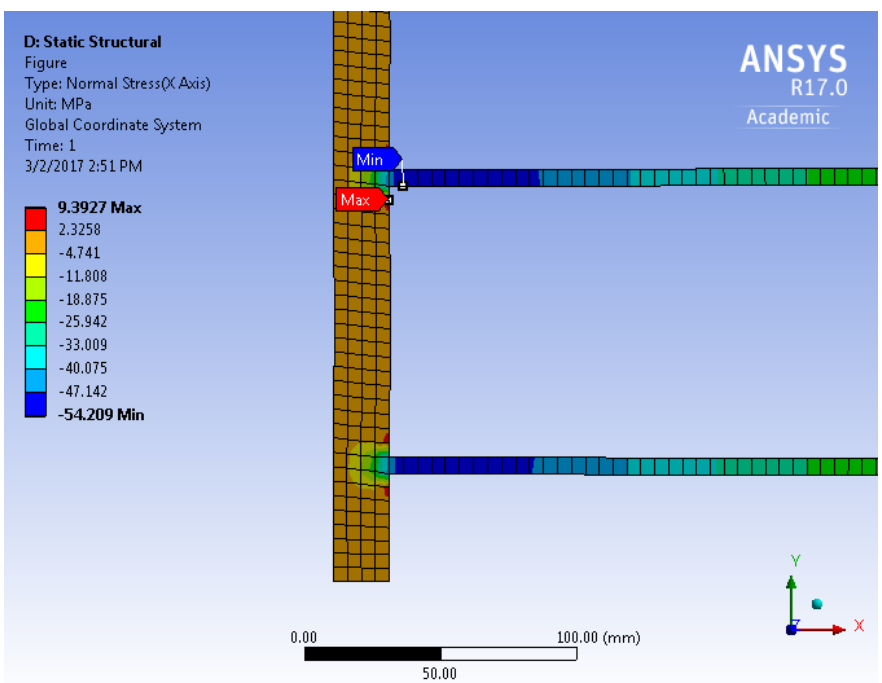

Figure A.30: Radial Stress, Static Structural with thermal load 2D (Fin length: 315 mm)

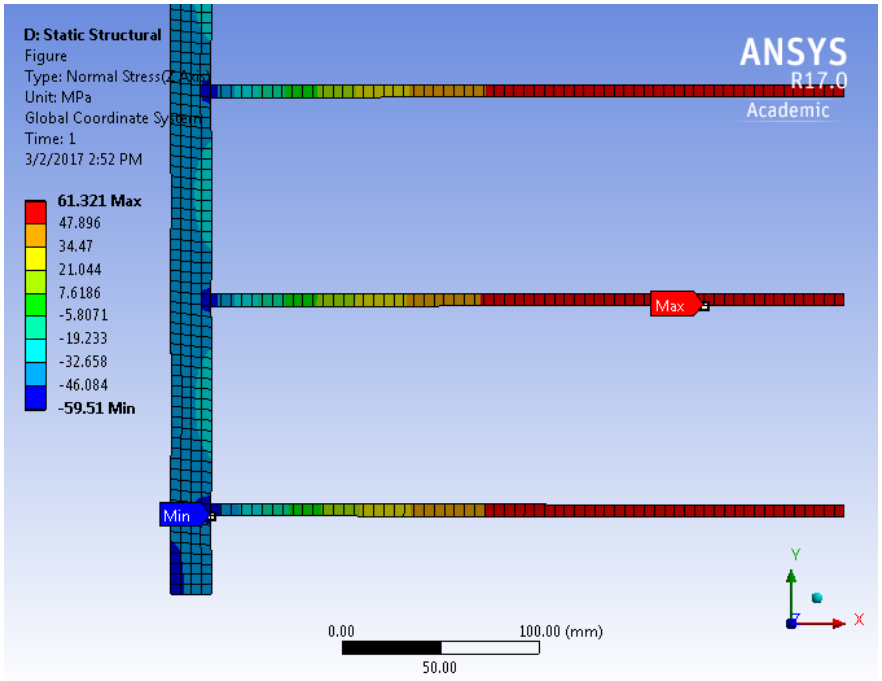

Figure A.31: Hoop Stress, Static Structural with thermal load 2D (Fin length: 315 mm)

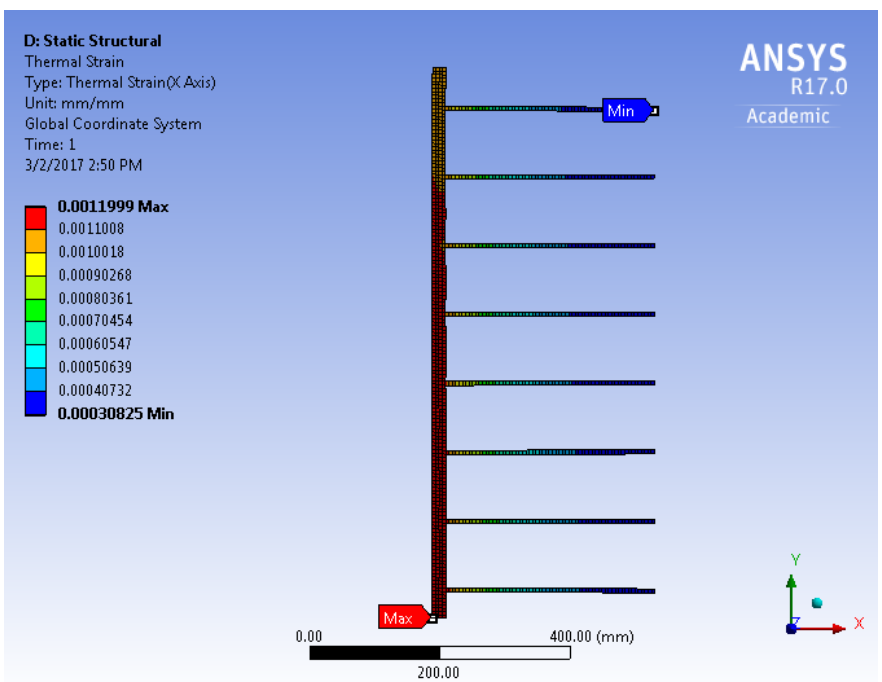

Figure A.32: Thermal Strain, Static Structural with thermal load 2D (Fin length: 315 mm)

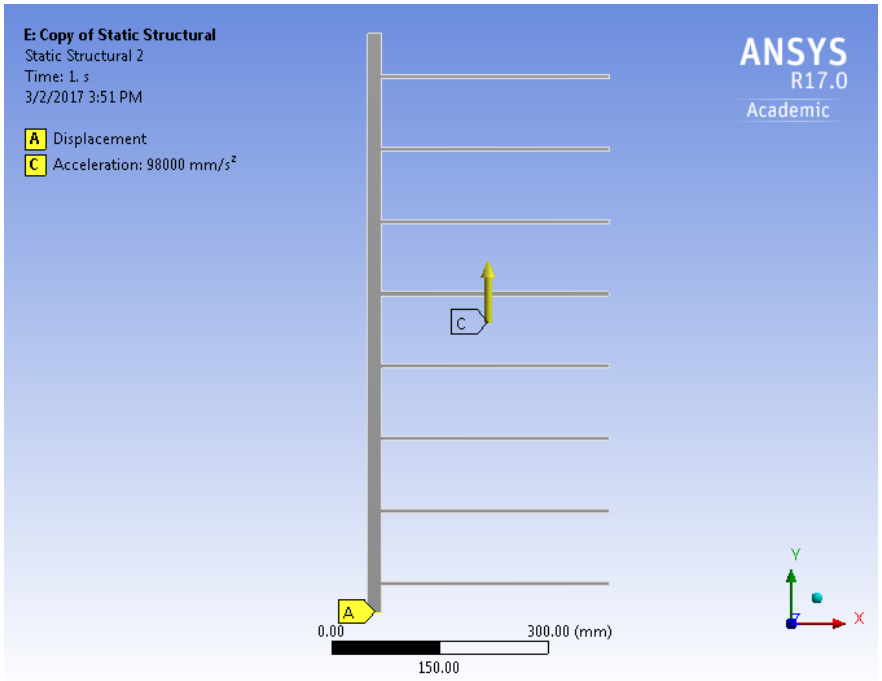

Figure A.33: Boundary Conditions, Static Structural, 10G acceleration 2D (Fin length: 315 mm)

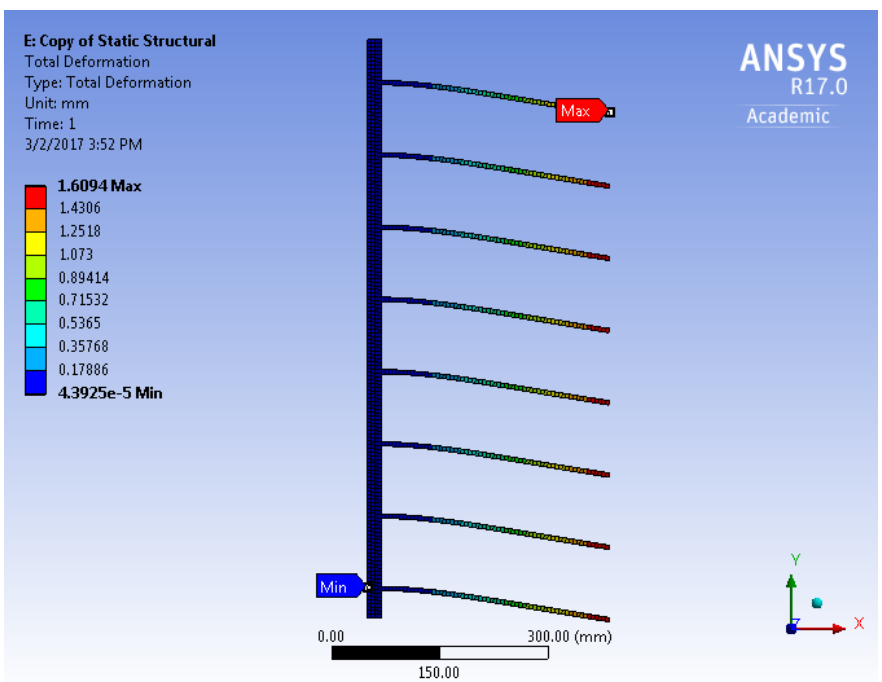

Figure A.34: Total Deformation, Static Structural, 10G acceleration 2D (Fin length: 315 mm)

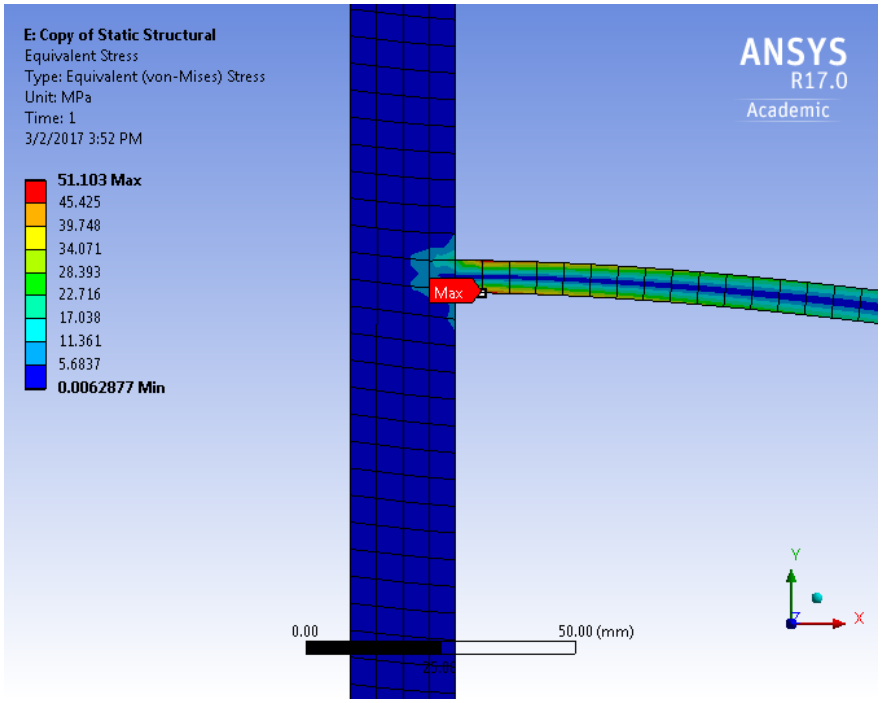

Figure A.35: Equivalent Stress, Static Structural, 10G acceleration 2D (Fin length: 315 mm)

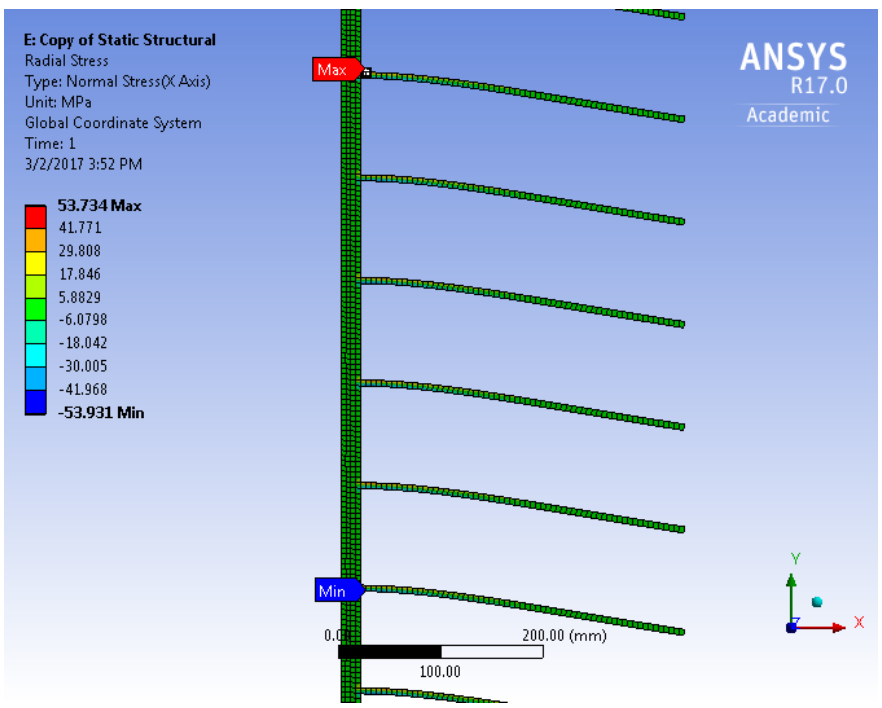

Figure A.36: Radial Stress, Static Structural, 10G acceleration 2D (Fin length: 315 mm)

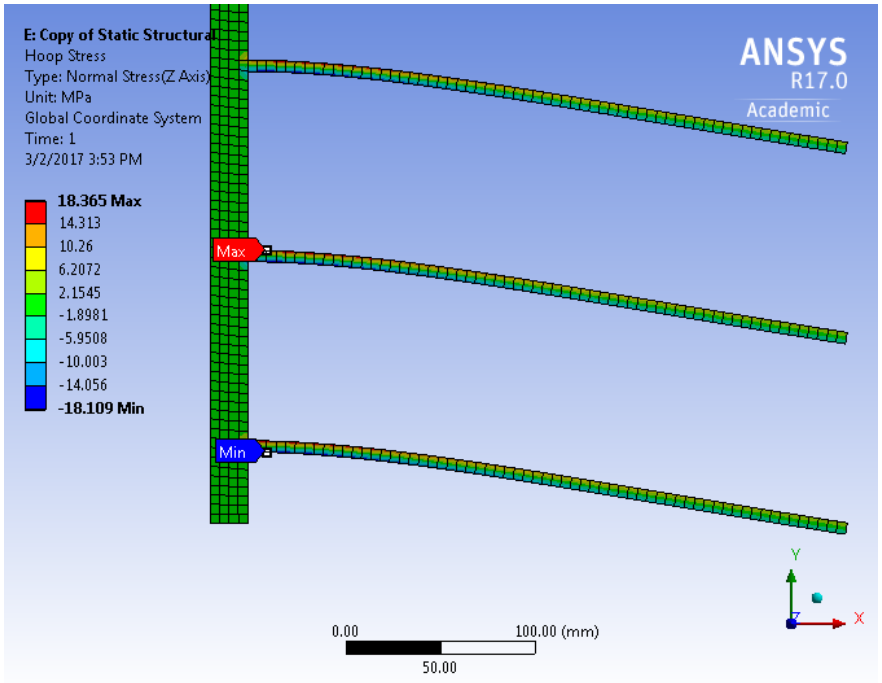

Figure A.37: Hoop Stress, Static Structural, 10G acceleration 2D (Fin length: 315 mm)

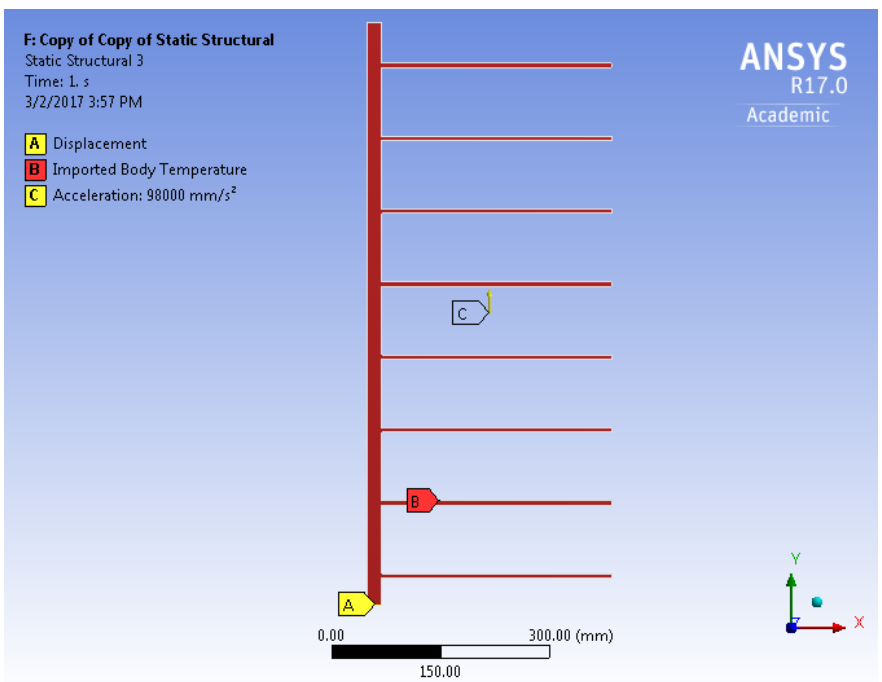

Figure A.38: Boundary Conditions, Static Structural, Solution Combination 2D (Fin length: 315 mm)

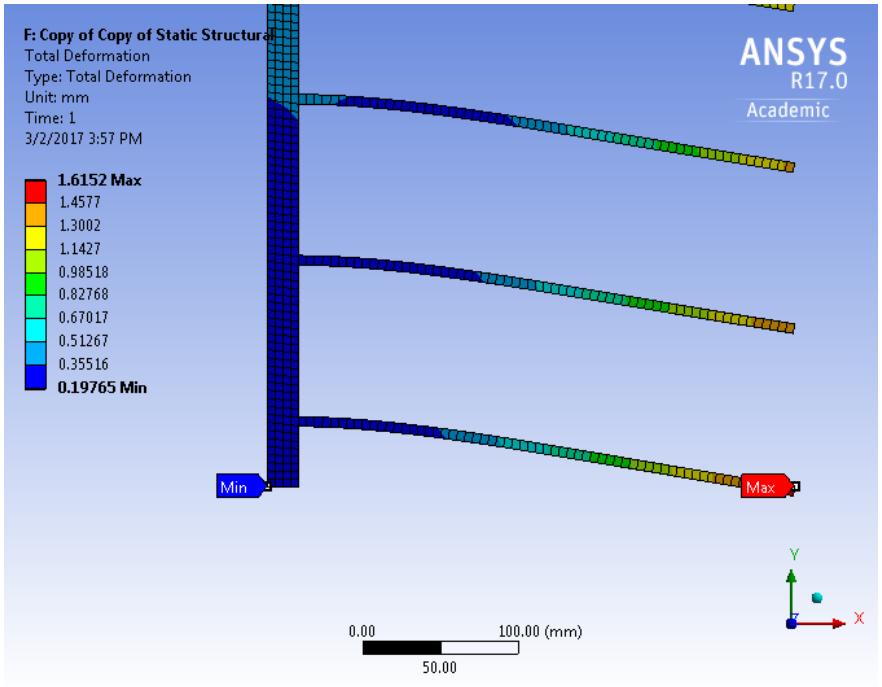

Figure A.39: Total Deformation, Static Structural, Solution Combination 2D (Fin length: 315 mm)

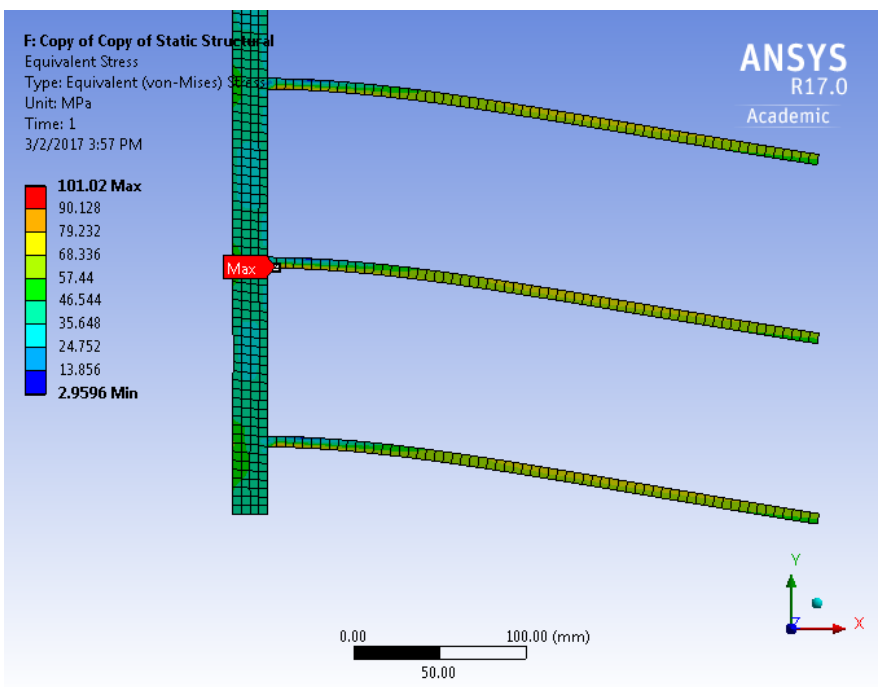

Figure A.40: Equivalent Stress, Static Structural, Solution Combination 2D (Fin length: 315 mm)

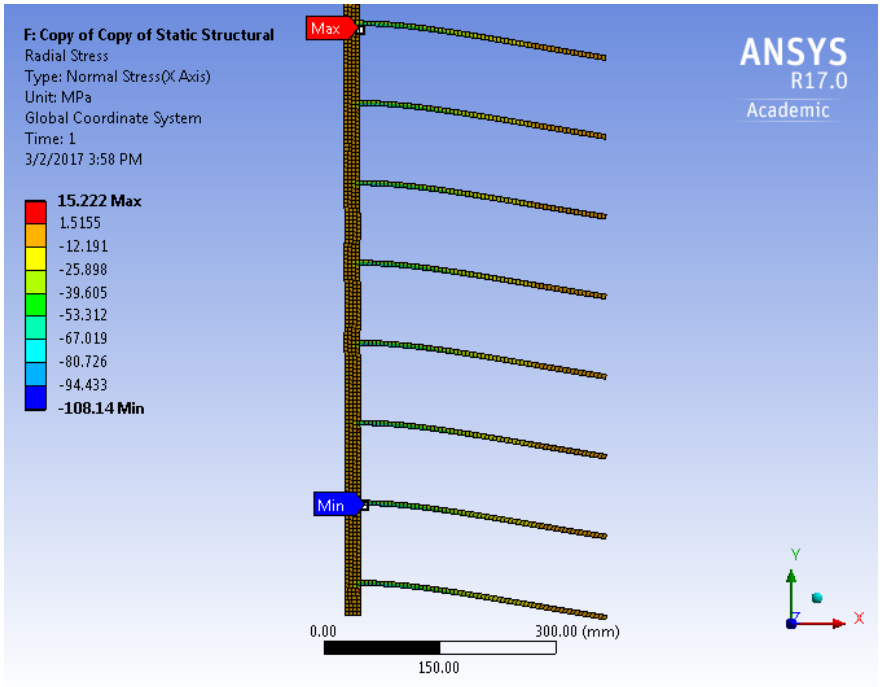

Figure A.41: Radial Stress, Static Structural, Solution Combination 2D (Fin length: 315 mm)

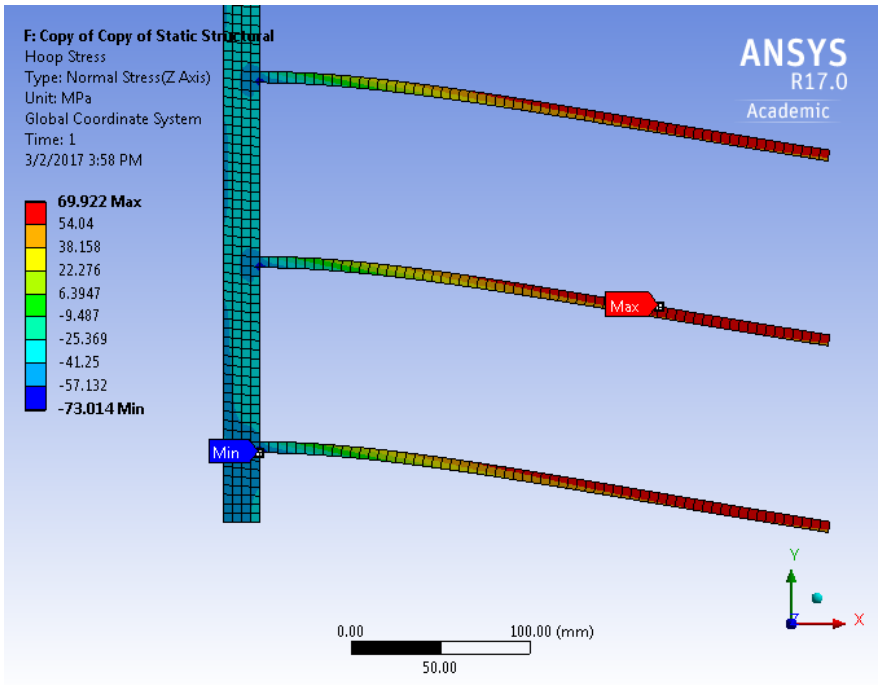

Figure A.42: Hoop Stress, Static Structural, Solution Combination 2D (Fin length: 315 mm)

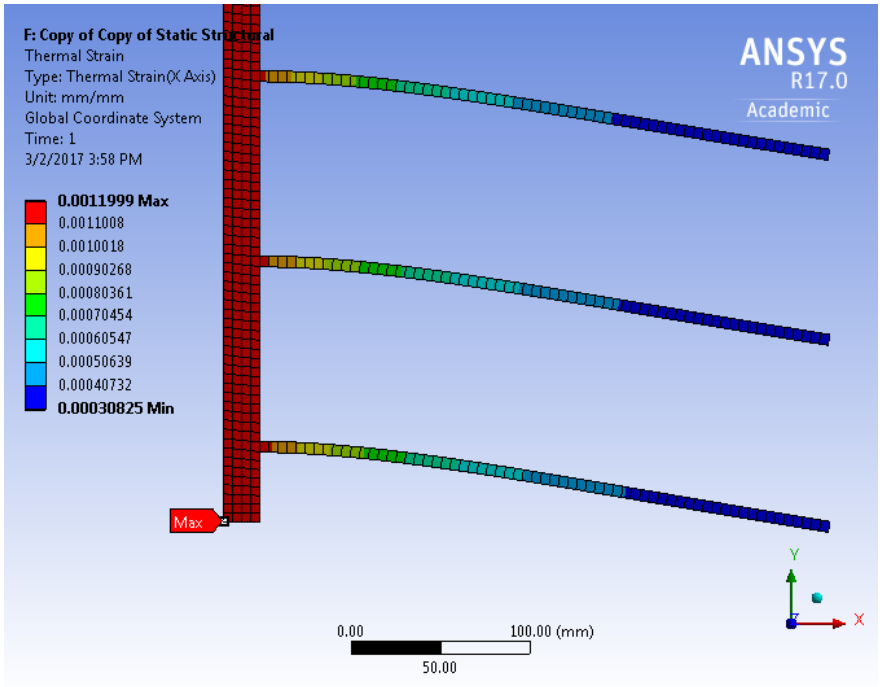

Figure A.43: Thermal Strain, Static Structural, Solution Combination 2D (Fin length: 315 mm)

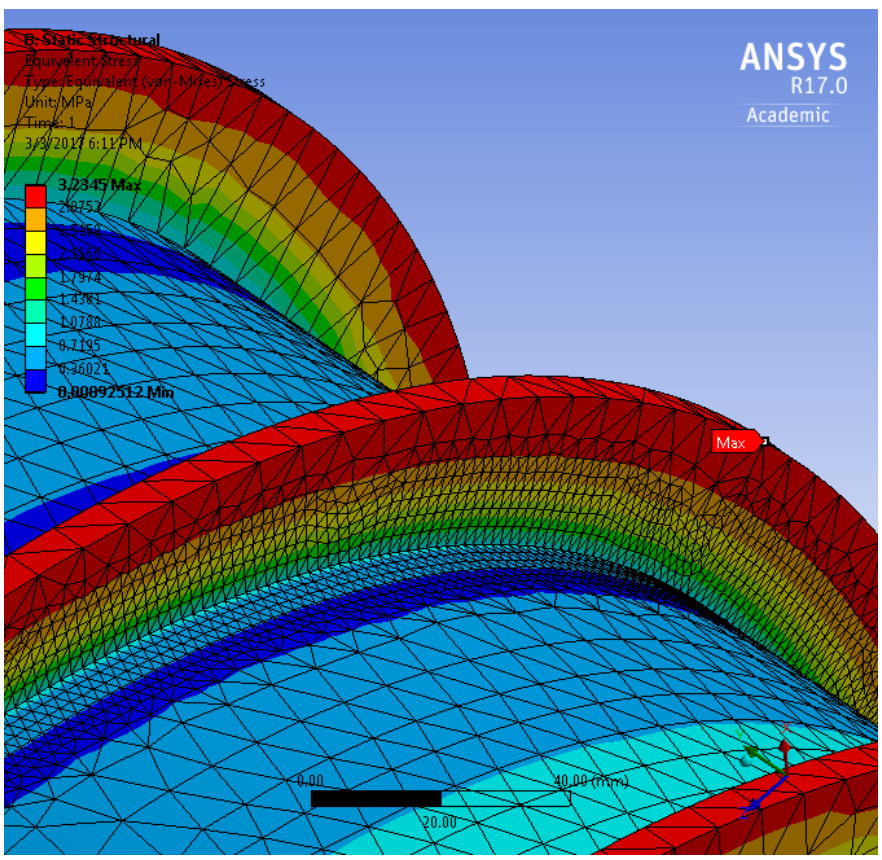

Figure A.44: Equivalent Stress, Static Structural with thermal load 3D, Refined Mesh (Fin length: 25 mm)

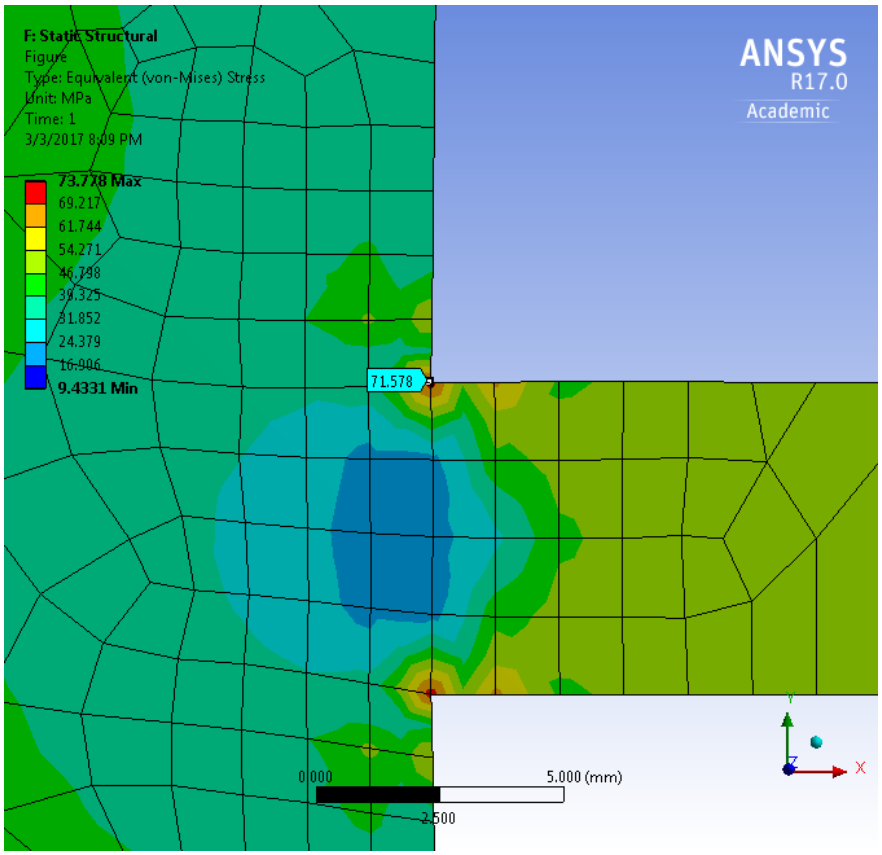

Figure A.45: Singularity for Static Structural Analysis 2D, Refinement=3 (Fin length=315 mm)