24-650 Applied Finite Element Analysis

Homework No 7
Beam-to-Beam Contact
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The objective of this assignment was to calculate the force-deflection stiffness ($K = F/\delta$) for several contact conditions for the geometry shown in Figure 1. The material used was structural steel.

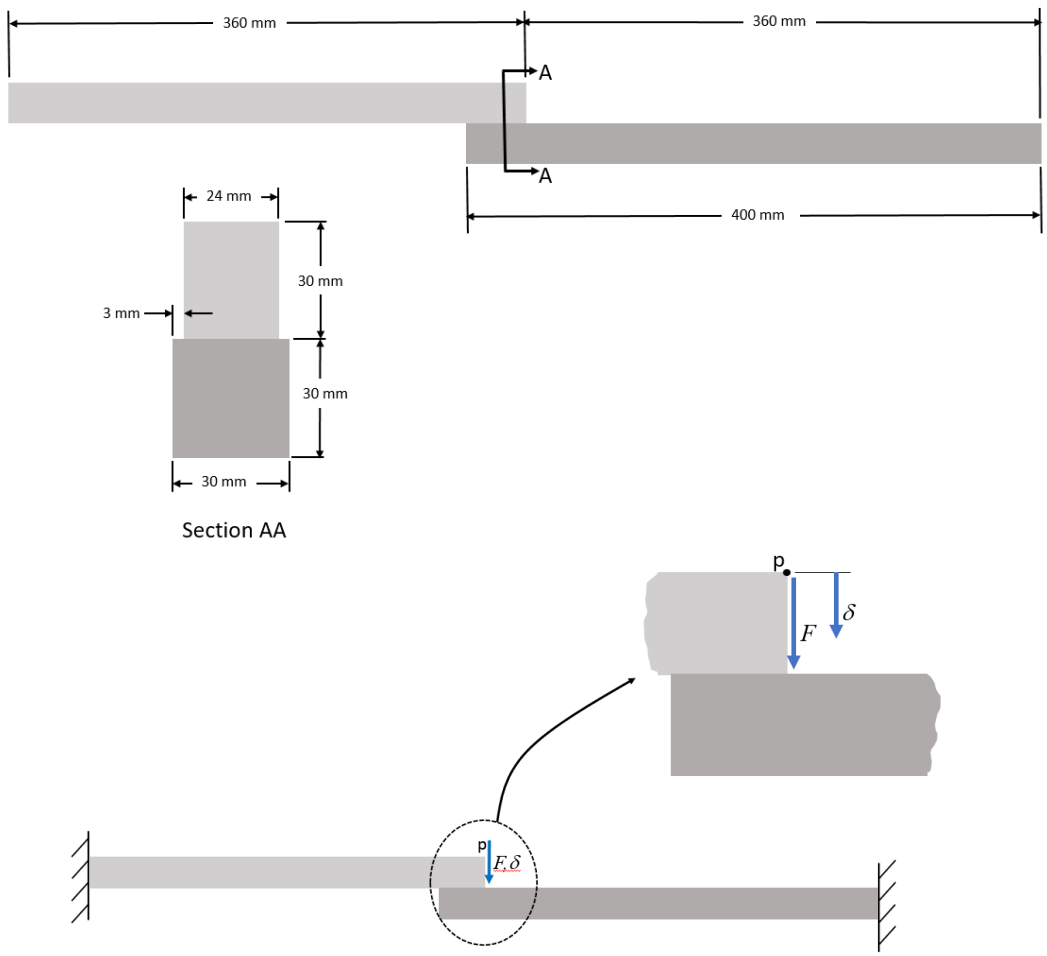


Figure 1: Beam-to-Beam contact

1. Setup

The first step was to create the geometry in SpaceClaim and then import it to Ansys Mechanical in a Static Structural module. Three different cases were analyzed:

- **Case 1:** No contact (Figure A.1)
- **Case 2:** No separation contact (Figure A.5)
- **Case 3:** Frictionless contact (Figure A.9)

The mesh used was the default one. The mesh consisted of **769 nodes** and **90 elements** for Cases 2 and 3, and **298 nodes** and **30 elements** for Case 1. This is shown in Figure A.2, Figure A.6.

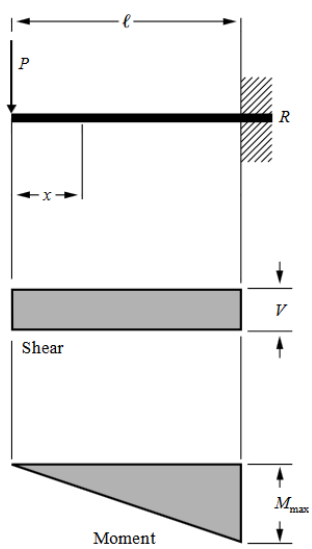
2. Results and Analysis

The results table is presented below.

Case #	Description	K, hand calc (N/mm)	K, FEA (N/mm)	Question
1	No contact (Ignore the bottom beam)	694.44	694.927	N/A
2	No Separation contact	6,250	5,919.96	For Behavior, is Program Controlled or Symmetric contact better and why?
3	Frictionless Contact	1,438.49	1,492.98	For Detection Method, is Program Controlled or Nodal-Normal contact better and why?

Table 1: Results

Case 1:



$$R = V \quad \ldots\ldots\ldots\ldots\ldots \quad = P$$

$$M_{max} \text{ (at fixed end)} \quad \ldots\ldots\ldots\ldots \quad = P\ell$$

$$M_x \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad = Px$$

$$\Delta_{max} \text{ (at free end)} \quad \ldots\ldots\ldots\ldots \quad = \frac{P\ell^3}{3EI}$$

$$\Delta_x \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad = \frac{P}{6EI}(2\ell^3 - 3\ell^2 x + x^3)$$

Figure 2: Cantilever Beam – Concentrated Load at Free End [1]

[1] Obtained from http://www.awc.org/pdf/codes-standards/publications/design-aids/AWC-DA6-BeamFormulas-0710.pdf

As can be seen in Figure 2, the max deformation of the beam for Case 1 is at the free end and is defined by (using the notation shown in Figure 1):

$$\delta_P = \frac{F \cdot {l_1}^3}{3 \cdot E \cdot I_1}$$

In this case:

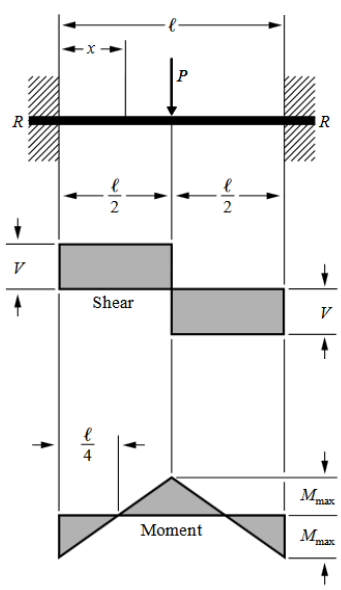
- $F = 1000\ [N]$
- $l_1 = 360\ [mm]$
- $E = 200\ [GPa]$
- $I_1 = \frac{1}{12} \cdot b_1 \cdot {h_1}^3 = \frac{1}{12} \cdot 24 \cdot 30^3 = 54{,}000\ [mm^4]$

$$\delta_P = \frac{F \cdot {l_1}^3}{3 \cdot E \cdot I_1} = 1.44\ [mm]$$
$$K = \frac{F}{\delta_P} = 694.44\ [N/mm]$$

This value is similar to the value obtained using Finite Elements, which was **694.927 [N/mm]**. The difference is about **0.07 %**, which is very small. The results confirm that the Cantilever Beam analytical expression is very accurate**.** These results are shown in Figure A.3 and Figure A.4.

Case 2:

For this case, we can make the assumption that the two beams with the bonded contact will have a deformation similar to one beam with an average Second Moment of Area value and with both ends fixed.



$$R = V \ldots\ldots\ldots\ldots\ldots = \frac{P}{2}$$

$$M_{max} \text{ (at center and ends)} \ldots\ldots = \frac{P\ell}{8}$$

$$M_x \left(\text{when } x < \frac{\ell}{2}\right) \ldots\ldots\ldots = \frac{P}{8}(4x - \ell)$$

$$\Delta_{max} \text{ (at center)} \ldots\ldots\ldots = \frac{P\ell^3}{192EI}$$

$$\Delta_x \left(\text{when } x < \frac{\ell}{2}\right) \ldots\ldots\ldots = \frac{Px^2}{48EI}(3\ell - 4x)$$

Figure 3: Beam Fixed at Both Ends- Concentrated Load at Center [2]

[2] Obtained from http://www.awc.org/pdf/codes-standards/publications/design-aids/AWC-DA6-BeamFormulas-0710.pdf

As can be seen in Figure 3, the max deformation of the beam for Case 2 is at the center and is defined by (using the notation shown in Figure 1):

$$\delta_P = \frac{F \cdot l^3}{192 \cdot E \cdot I}$$

In this case:

- $F = 1000\ [N]$
- $l = 360 + 360 = 720\ \ [mm]$
- $E = 200\ [GPa]$
- $I = \frac{I_1 + I_2}{2} = \frac{\frac{1}{12} \cdot b_1 \cdot {h_1}^3 + \frac{1}{12} \cdot b_2 \cdot {h_2}^3}{2} = \frac{\frac{1}{12} \cdot 24 \cdot 30^3 + \frac{1}{12} \cdot 30 \cdot 30^3}{2} = 60{,}750\ [mm^4]$

$$\delta_P = \frac{F \cdot l^3}{3 \cdot E \cdot I} = 0.16\ [mm]$$
$$K = \frac{F}{\delta_P} = 6{,}250\ [N/mm]$$

This value is similar to the value obtained using Finite Elements, which was **5,919.96 [N/mm]**. The difference is about **5.57 %** which is good enough for the assumptions made. These results are shown in Figure A.7 and Figure A.8.

The results shown were obtained using the Contact Behavior as **Program Controlled**, which by default is **Asymmetric contact**. For that contact, there is one surface that has the Contact Nodes and a second surface that is the Target Surface. The actual contact takes place in the Target surface. This behavior is good enough for most of the cases, but there can be situations were using Asymmetric contact can produce penetration between the surfaces. For those cases, it is better to use a **Symmetric contact**, which considers Contact Nodes and Target Surface in both surfaces. Using the Symmetric contact, the value for total deformation at the point P was **0.15932 [mm]** (shown in Figure A.12), which is similar to **0.16892 [mm]** obtained with the Program Controlled behavior (shown in Figure A.8). In Figure A.14 and Figure A.15 the penetration is shown for both contact behaviors. It can be seen that for both, the penetration is negligible. In situations where it is difficult to identify which surface should be the contact or the target, it is always better to use Symmetric contact. Other advantage of Symmetric contact is that the results are available in both surfaces.

Case 3:

For Case 3, we can take Case 1, and add the second beam with a Reaction Force “R” as shown in Figure 4.

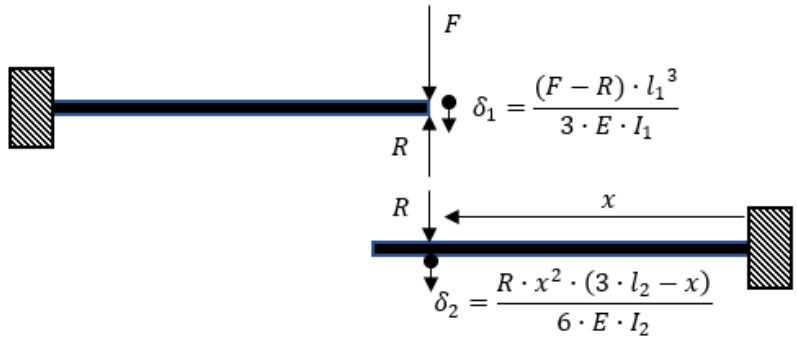


Figure 4: Hand Calculation Diagram for Case 3

As can be seen in Figure 4, we can assume that the deformation of the point of contact for both beams is the same, so we can say that:

$$\delta_1 = \delta_2$$

$$\frac{(F-R)\cdot {l_1}^3}{3\cdot E\cdot I_1} = \frac{R\cdot x^2\cdot(3\cdot l_1 - x)}{6\cdot E\cdot I_2}$$

So,

$$R = \frac{F}{1 + \frac{I_1\cdot x^2\cdot(3\cdot l_1 - x)}{2\cdot I_2\cdot {l_2}^3}}$$

In this case:

- $F = 1000\ [N]$
- $l_1 = 360\ [mm]$
- $l_2 = 400\ [mm]$
- $x = 40\ [mm]$
- $E = 200\ [GPa]$
- $I_1 = \frac{1}{12}\cdot b_1\cdot {h_1}^3 = \frac{1}{12}\cdot 24\cdot 30^3 = 54{,}000\ [mm^4]$
- $I_2 = \frac{1}{12}\cdot b_2\cdot {h_2}^3 = \frac{1}{12}\cdot 30\cdot 30^3 = 67{,}500\ [mm^4]$

$$R = \frac{F}{1 + \frac{I_1\cdot x^2\cdot(3\cdot l_1 - x)}{2\cdot I_2\cdot {l_2}^3}} = 517.24\ [N]$$

$$\delta_P = \delta_1 = \delta_2 = \frac{(F - R) \cdot {l_1}^3}{3 \cdot E \cdot I_1} = 0.695\ [mm]$$

$$K = \frac{F}{\delta_P} = 1{,}438.49\ [N/mm]$$

This value is similar to the value obtained using Finite Elements, which was **1,492.98 [N/mm]**. The difference is about **3.64 %** which is good enough for the assumptions made. These results are shown in Figure A.10 and Figure A.11.

The results shown were obtained using the Detection Method as **Program Controlled**, which uses integration point detection (On Gauss Point), which is has a very good convergence behavior (depending on the penetration) and is useful for any contact behavior. The down side is that if you have a lot of curvature or even a corner, there can be a big penetration between surfaces. The **Normal-Nodal from Contact** method detects contact at nodes. This a more expensive method and requires more iterations for equilibrium if chattering is present. The advantage of this method is that usually, penetration is near-zero and the user don't need to change values for the Normal Stiffness. In this case, the total deformation obtained with the Normal-Nodal from Contact was **0.64003 [mm]** and **0.66979 [mm]** for the Program Controlled method. Those results are shown in Figure A.11 and Figure A.13. The penetration for both cases are shown in Figure A.16 and Figure A.17. It can be seen that for both methods, the value is near-zero, so it doesn't make any difference to use any method.

3. Appendix

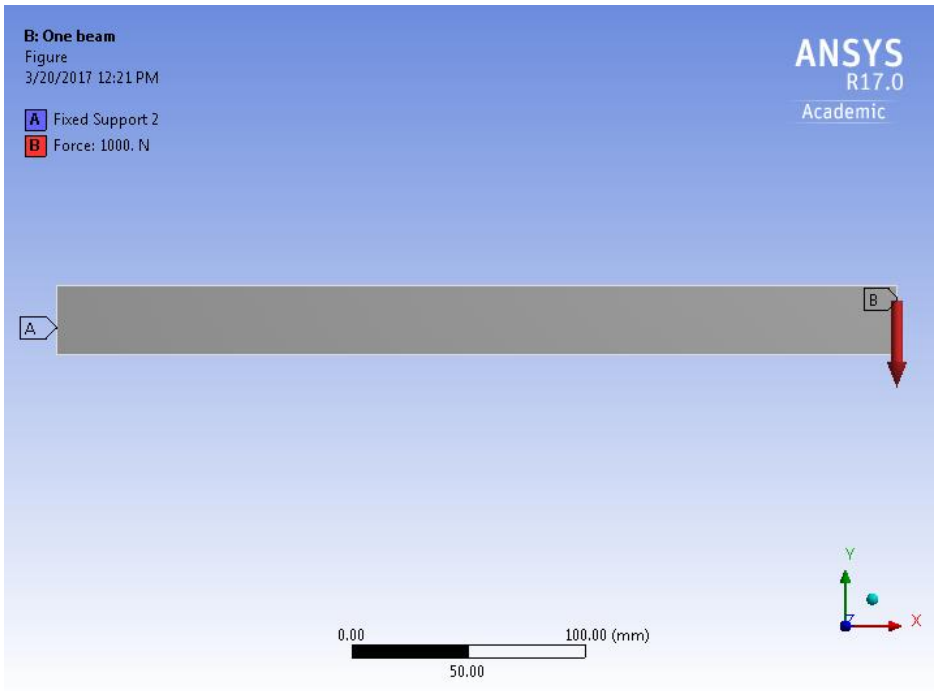


Figure A.1: Boundary Conditions, Case 1

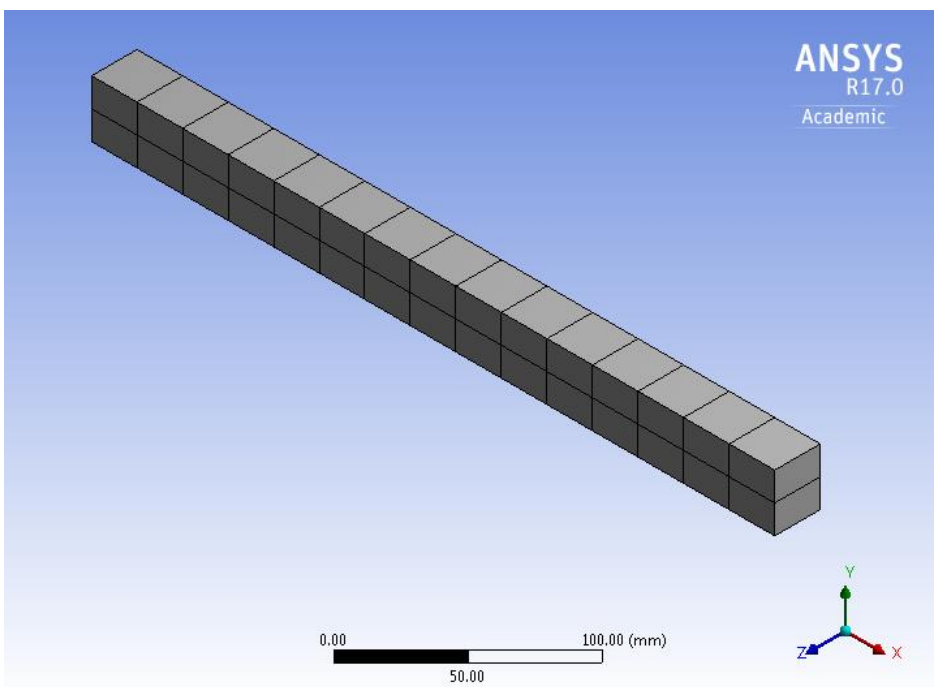


Figure A.2: Default Mesh, Case 1

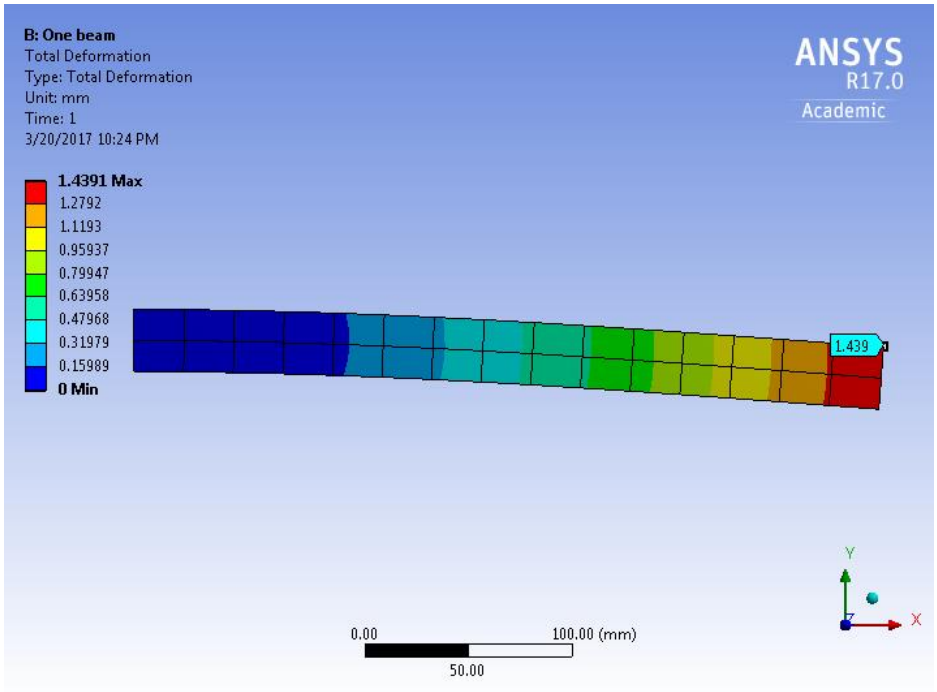


Figure A.3: Total Deformation, Case 1

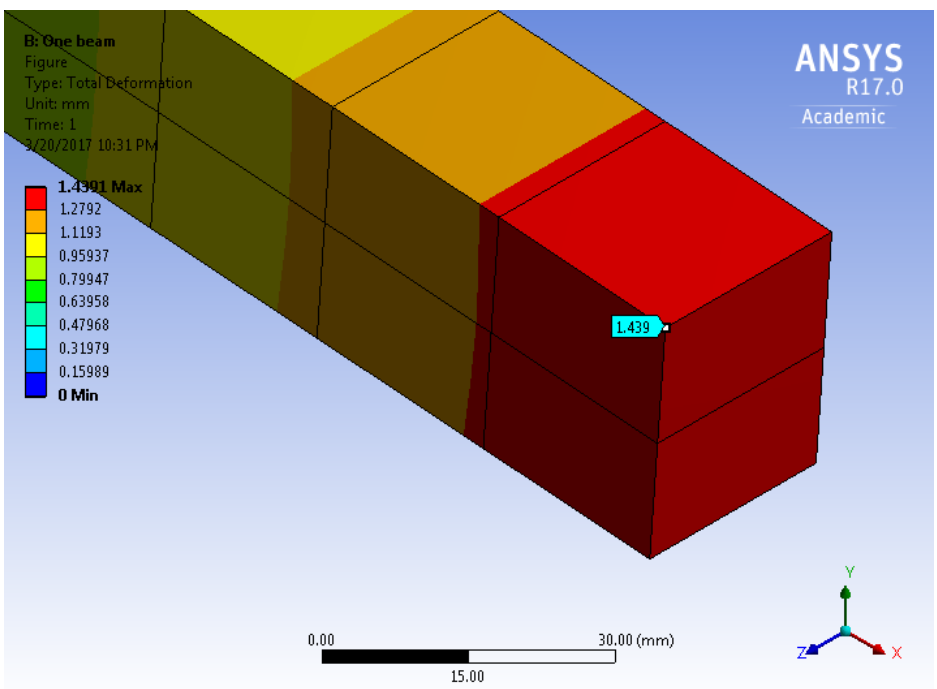


Figure A.4: Total Deformation, Zoom, Case 1

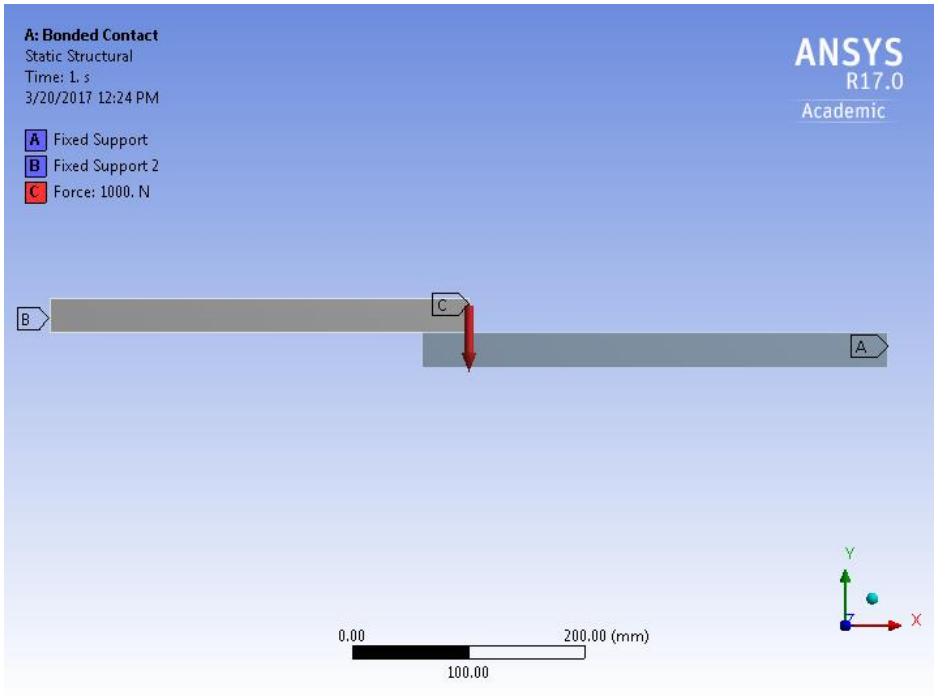


Figure A.5: Boundary Conditions, Case 2

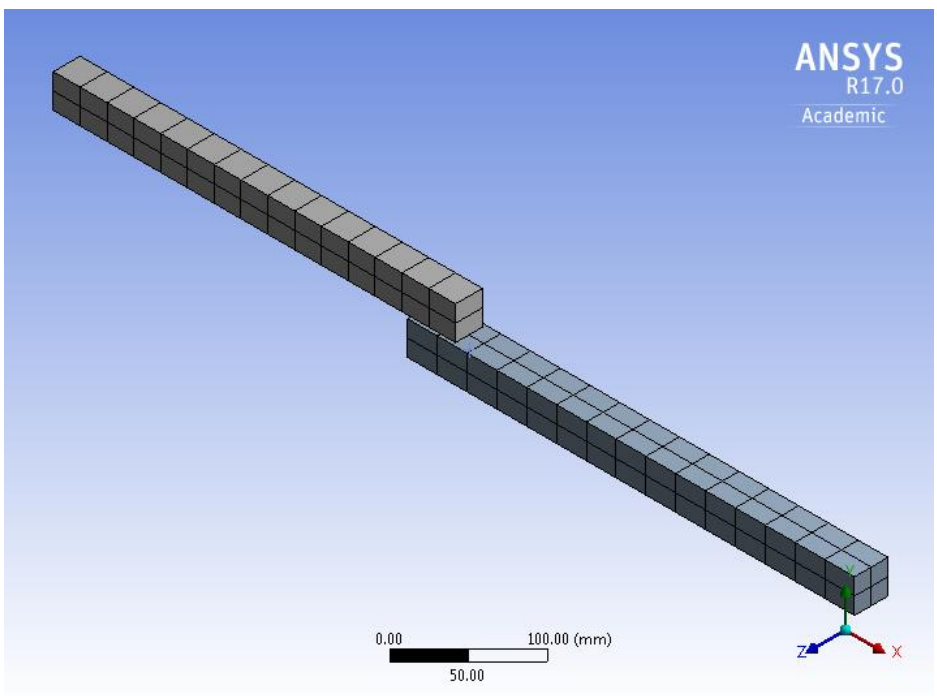


Figure A.6: Default Mesh, Case 2 and Case 3

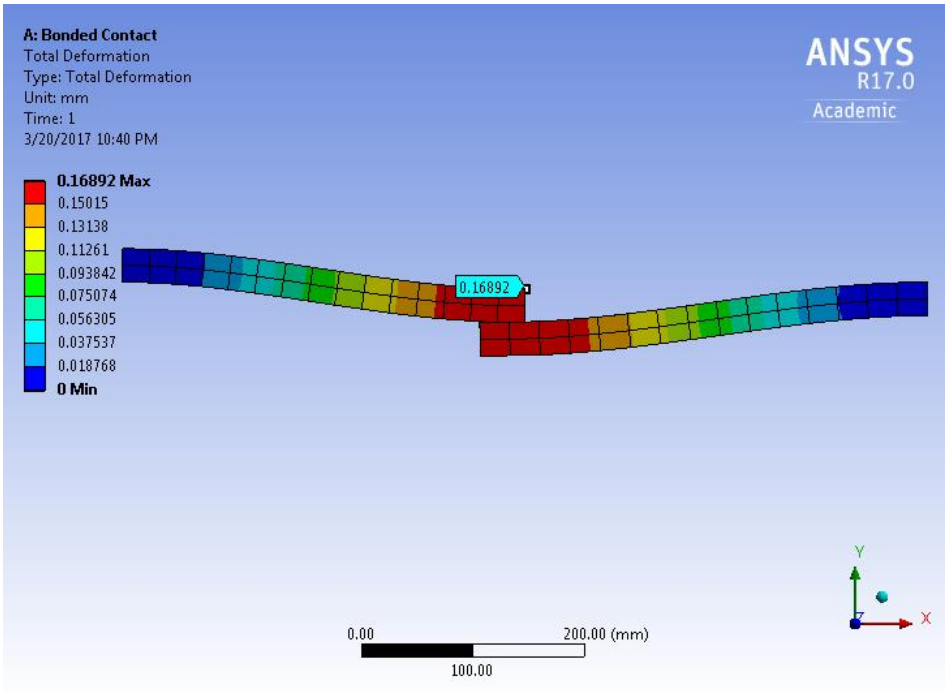


Figure A.7: Total Deformation, Case 2

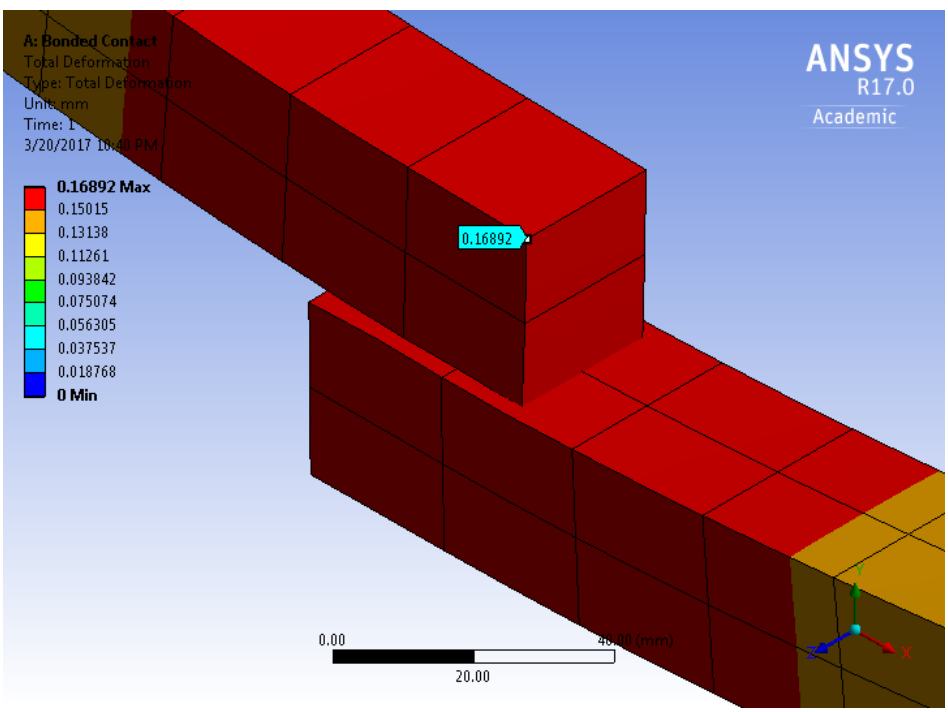


Figure A.8: Total Deformation, Zoom, Case 2

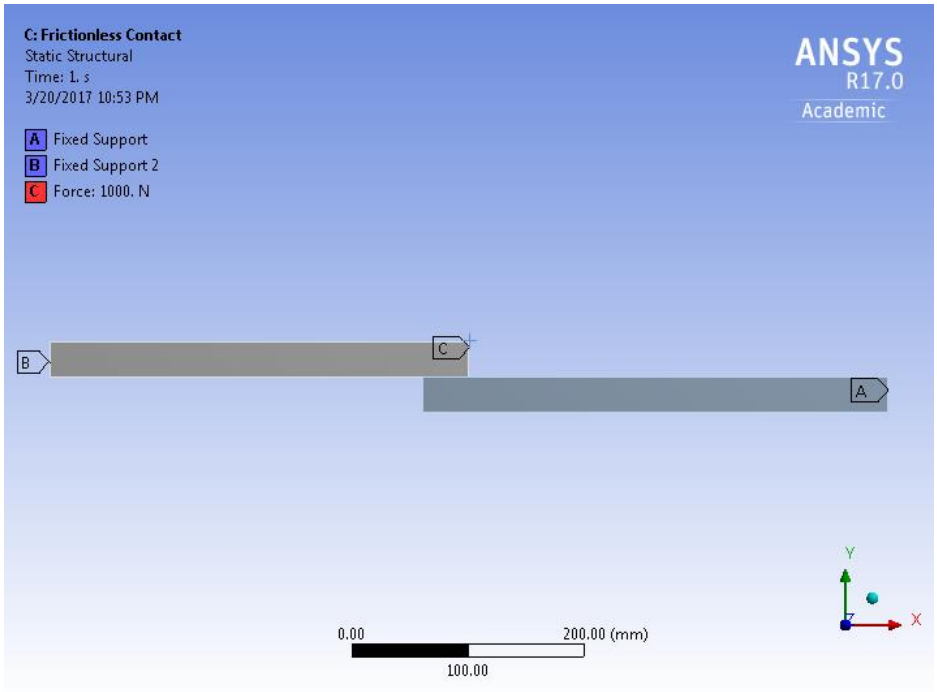


Figure A.9: Boundary Conditions, Case 3

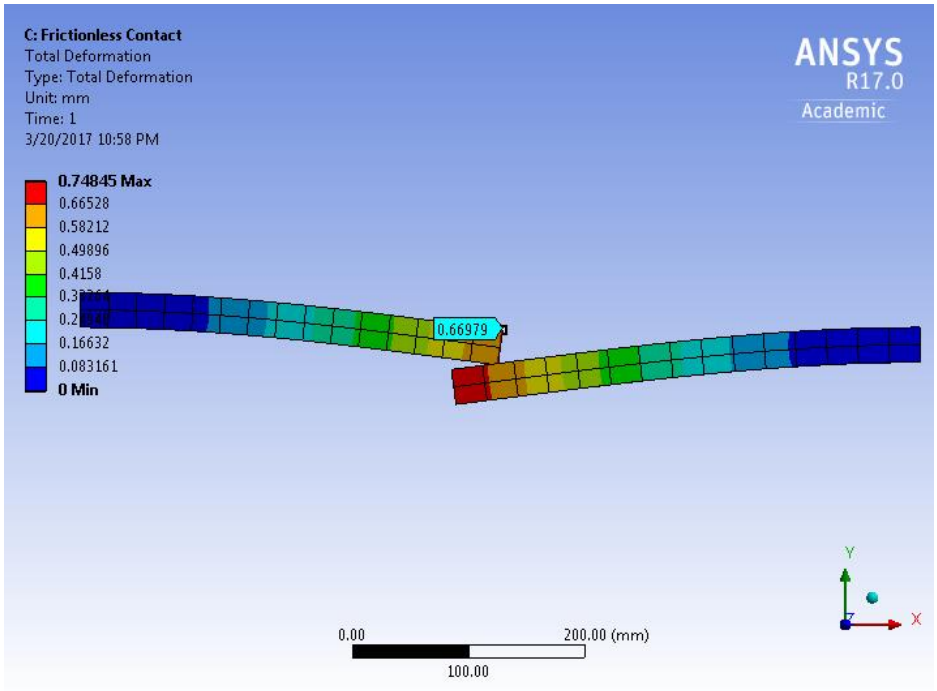


Figure A.10: Total Deformation, Case 3

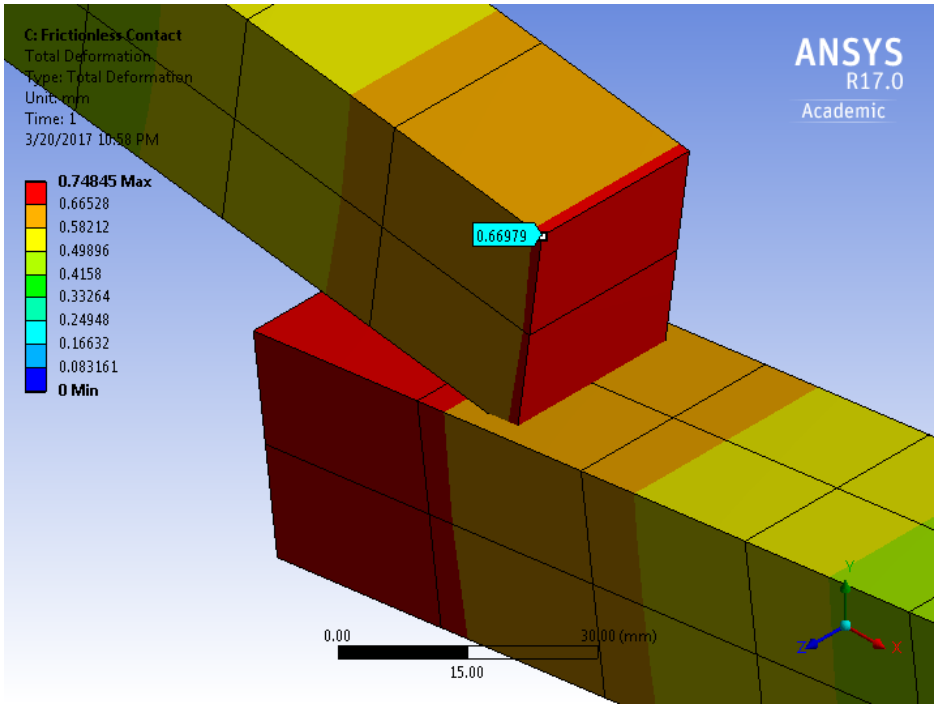


Figure A.11: Total Deformation, Zoom, Case 3

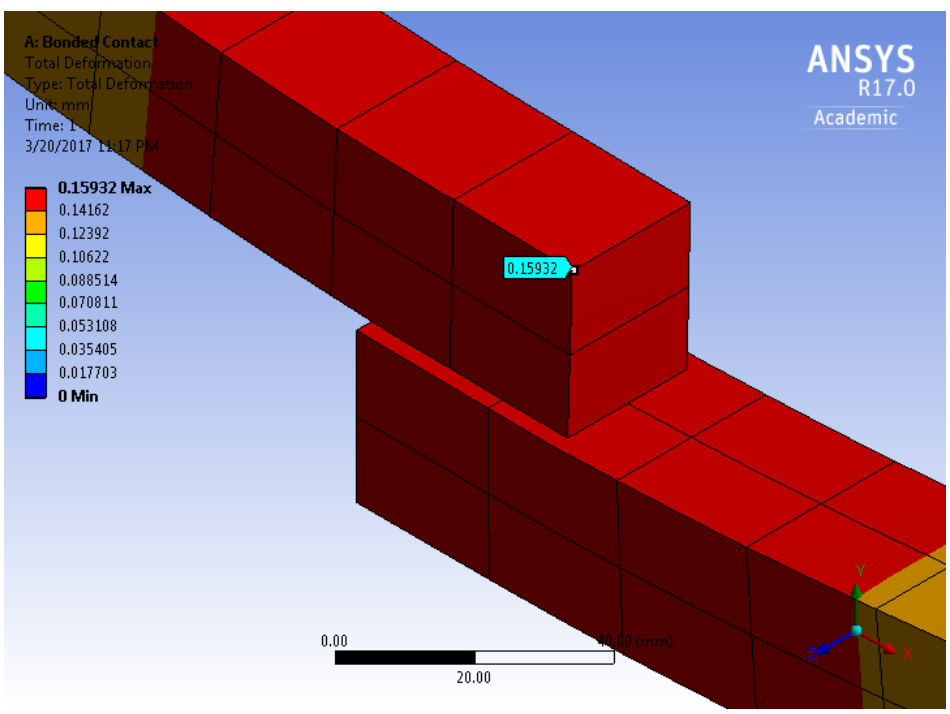


Figure A.12: Total Deformation, Zoom, Contact Behavior: Symmetric, Case 2

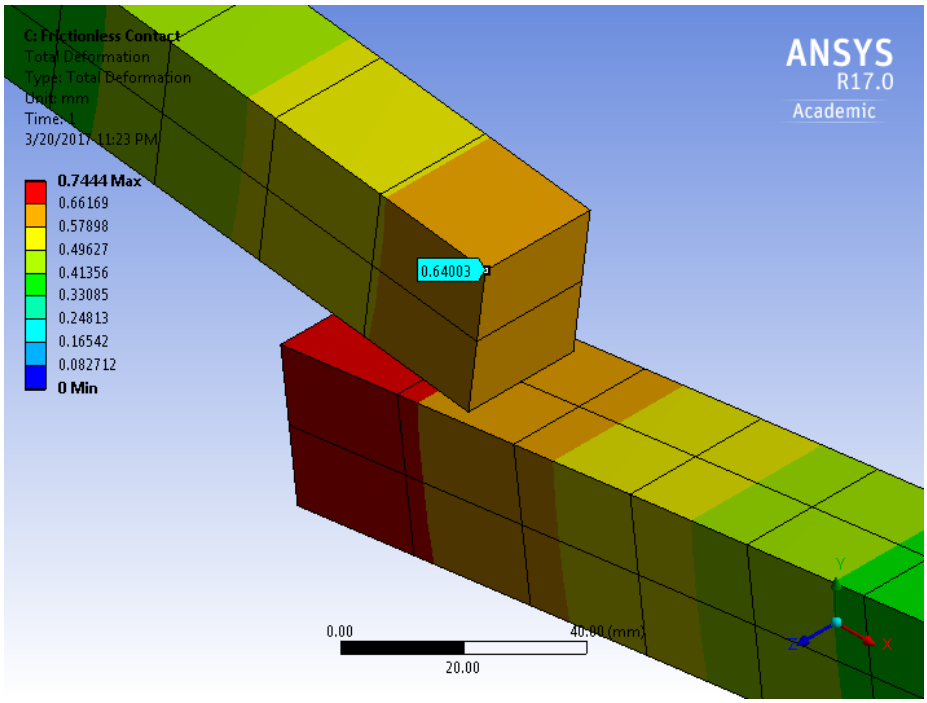


Figure A.13: Total Deformation, Zoom, Detection Method: Nodal-Normal contact, Case 2

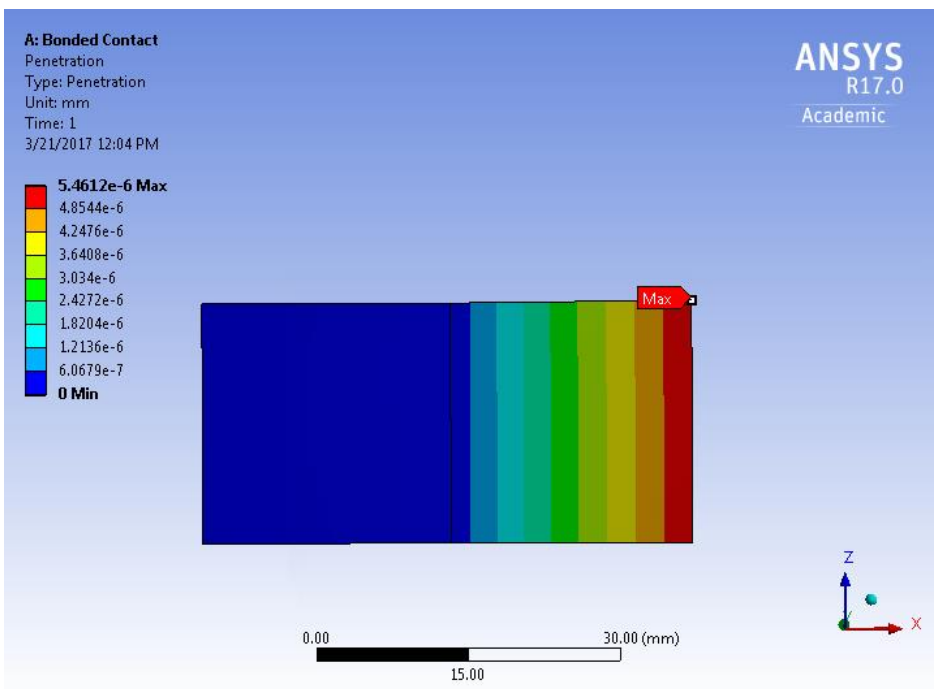


Figure A.14: Bonded Contact, Penetration, Contact Behavior: Program Controlled, Case 2

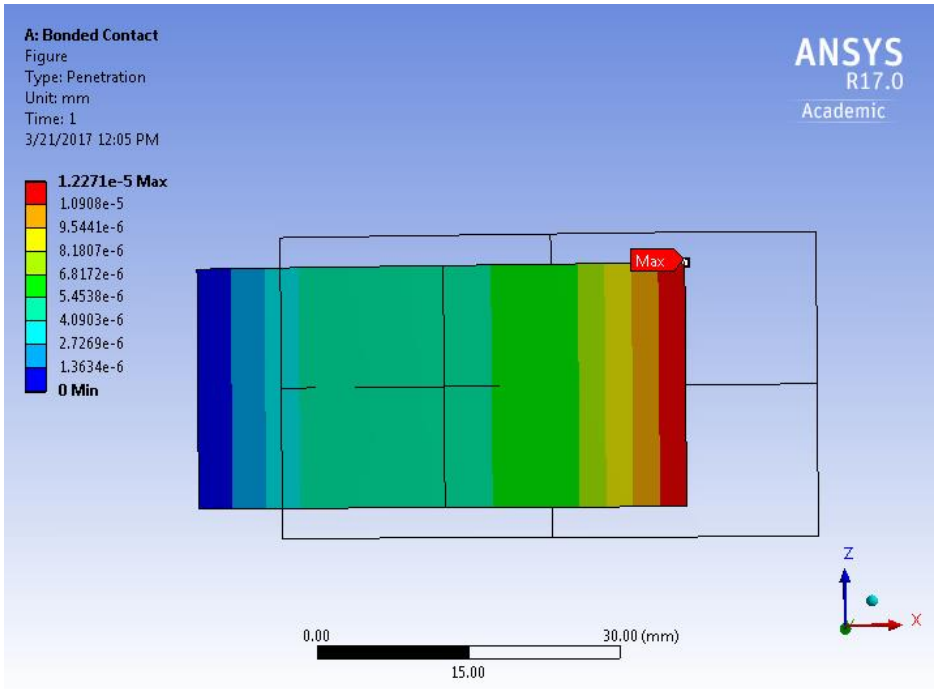


Figure A.15: Bonded Contact, Penetration, Contact Behavior: Symmetric, Case 2

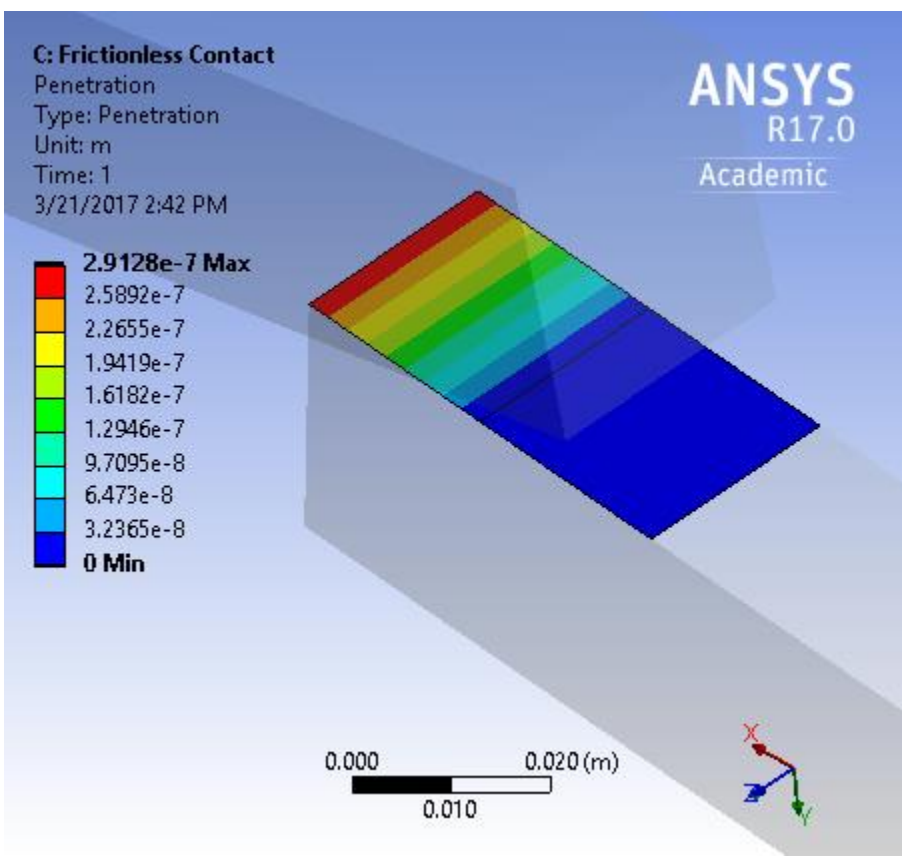


Figure A.16: Frictionless Contact, Penetration, Detection Method: Program Controlled, Case 3

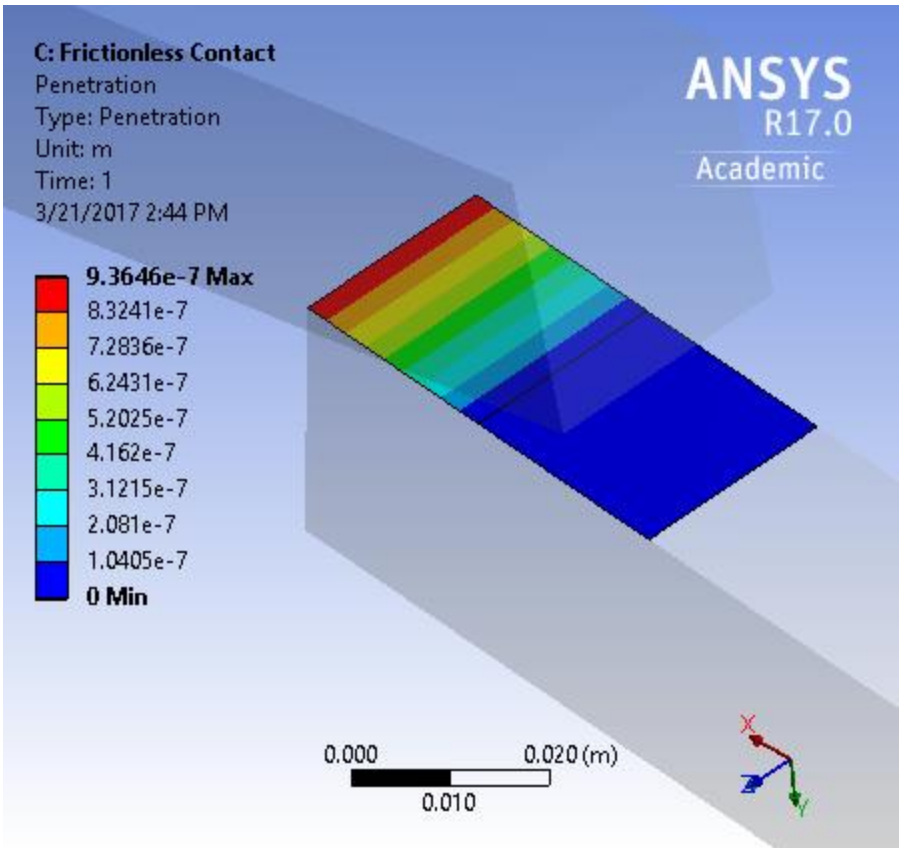


Figure A.17: Frictionless Contact, Penetration, Detection Method: Nodal-Normal from contact, Case 3